Global and local minimizers for a $p$-Ginzburg Landau type energy in the plane

November 1, 2007

Abstract

We consider the functional

$$E(u) = \int_{\mathbb{R}^2} |\nabla u|^p + (1 - |u|^2)^2,$$

for $p > 2$. We prove the existence of a global minimizer for this functional in the space of functions whose energy is bounded and their degree at “infinity” is one. The same result is proved in $\mathbb{R}_+^2$ in some right semi-neighborhood of $p = 2$.

We then discuss the minimizer in the class of radially symmetric functions. Among other things we prove its uniqueness, show it is locally stable in some right semi-neighborhood of $p = 2$, and obtain its limit as $p \to \infty$.

These results have been obtained in collaboration with L. Berlyand, D. Golovaty and I. Shafrir.