Title: Pairs of Dual Wavelet Frames and Riesz Wavelets in Sobolev Spaces

Abstract: Wavelet frames and Riesz wavelets in Sobolev spaces are of interest in numerical algorithms and image processing. The traditional approach is to obtain wavelets in $L_2(R^d)$, and then to extend such wavelets to certain Sobolev spaces. This approach excludes many interesting wavelets in Sobolev spaces. In this talk, using a direct approach, we shall present a natural framework to study dual wavelet frames and Riesz wavelets in a pair of Sobolev spaces $(H^s(R^d), H^{-s}(R^d))$ for any real number s. We extend the mixed extension principle for pairs of dual wavelet frames from $L_2(R^d)$ to Sobolev spaces. In our construction, the smoothness and vanishing moments play separate and quite different roles in the primal and dual wavelet systems: The primal requires smoothness but no vanishing moments, while the dual required vanishing moments but low smoothness (may not in $L_2(R^d)$). As an example, we show that $\{2^{j(1/2-s)}B_m(2^j \cdot -k) \ : \ j \in N_0, k \in Z\}$ is a wavelet frame in $H^s(R)$ for any $0 < s < m - 1/2$, where B_m is the B-spline of order m. This is also true for a large class of refinable functions (no stability is required) including almost all box splines in any dimension. We further obtain and characterize dual Riesz wavelets in Sobolev spaces. For example, we show that any interpolatory wavelet system generated by an interpolatory refinable function $\phi \in H^s(R)$ with $s > 1/2$, which was considered by Donoho, is a Riesz basis of the Sobolev space $H^s(R)$. Our approach also naturally leads to a characterization of the Sobolev norm of a function in terms of weighted norm of its wavelet coefficient sequence. This talk is based on joint works with Z. Shen, in particular, [B. Han and Z. Shen, Dual wavelet frames and Riesz bases in Sobolev spaces, *Constr. Approx.*, to appear]. Our work is partially motivated by joint work with R. Q. Jia on Riesz wavelets in $L_2(R^d)$, joint work with I. Daubechies, A. Ron and Z. Shen on wavelet frames in $L_2(R^d)$, as well as by many other related works in the literature.