Title: Pairs of Dual Wavelet Frames and Riesz Wavelets in Sobolev Spaces

Abstract: Wavelet frames and Riesz wavelets in Sobolev spaces are of interest in numerical algorithms and image processing. The traditional approach is to obtain wavelets in $L_2(R^d)$, and then to extend such wavelets to certain Sobolev spaces. This approach excludes many interesting wavelets in Sobolev spaces. In this talk, using a direct approach, we shall present a natural framework to study dual wavelet frames and Riesz wavelets in a pair of Sobolev spaces $(H^s(R^d), H^{-s}(R^d))$ for any real number $s$. We extend the mixed extension principle for pairs of dual wavelet frames from $L_2(R^d)$ to Sobolev spaces. In our construction, the smoothness and vanishing moments play separate and quite different roles in the primal and dual wavelet systems: The primal requires smoothness but no vanishing moments, while the dual required vanishing moments but low smoothness (may not in $L_2(R^d)$). As an example, we show that $\{2^{j(1/2-s)}B_m(2^j \cdot -k) : j \in \mathbb{N}_0, k \in \mathbb{Z}\}$ is a wavelet frame in $H^s(R)$ for any $0 < s < m - 1/2$, where $B_m$ is the $B$-spline of order $m$. This is also true for a large class of refinable functions (no stability is required) including almost all box splines in any dimension. We further obtain and characterize dual Riesz wavelets in Sobolev spaces. For example, we show that any interpolatory wavelet system generated by an interpolatory refinable function $\phi \in H^s(R)$ with $s > 1/2$, which was considered by Donoho, is a Riesz basis of the Sobolev space $H^s(R)$. Our approach also naturally leads to a characterization of the Sobolev norm of a function in terms of weighted norm of its wavelet coefficient sequence. This talk is based on joint works with Z. Shen, in particular, [B. Han and Z. Shen, Dual wavelet frames and Riesz bases in Sobolev spaces, Constr. Approx., to appear]. Our work is partially motivated by joint work with R. Q. Jia on Riesz wavelets in $L_2(R^d)$, joint work with I. Daubechies, A. Ron and Z. Shen on wavelet frames in $L_2(R^d)$, as well as by many other related works in the literature.

Title: $C^\infty$ Symmetric Tight Wavelet Frames and Nonstationary Cascade Algorithms

Abstract: Motivated by the interesting work of Cohen-Dyn on nonstationary orthonormal wavelets, in this talk, we shall discuss nonstationary tight wavelet frames and convergence of nonstationary cascade algorithms in $L_2(R)$. We present a general algorithm for constructing nonstationary tight wavelet frames and show that there are compactly supported $C^\infty$ symmetric tight wavelet frames with the spectral frame approximation order. Furthermore, we present a family of symmetric compactly supported $C^\infty$ orthonormal complex wavelets in $L_2(R)$. A comprehensive analysis of nonstationary tight wavelet frames and orthonormal wavelet bases in $L_2(R)$ is given. We show that a Sobolev space of an arbitrary fixed order of smoothness can be characterized in terms of the weighted $\ell_2$-norm of the analysis wavelet coefficient sequences using a fixed compactly supported nonstationary tight wavelet frame in $L_2(R)$ derived from masks of pseudo-splines. Therefore, our constructed compactly supported nonstationary tight wavelet frames of $L_2(R)$ can be properly normalized into a pair of dual wavelet frames in any Sobolev space. This talk is based on [B. Han and Z. Shen, Compactly supported symmetric $C^\infty$ wavelets with spectral approximation order, SIAM J. Math. Anal., to appear] and [B. Han and Z. Shen, Characterization of Sobolev spaces of arbitrary smoothness using nonstationary tight wavelet frames, Israel J. Math., to appear].

Title: Adaptive Wavelet Scheme for an Image Denoising Model

Abstract: A popular method for image denoising is the total-variation-based Rudin-Osher-Fatemi model and its variants. Using a nonlinear diffusion process to preserve sharp edges while smoothing out the noise in the image, such models employ a highly nonlinear partial differential equation. In this talk, based on the framework of adaptive wavelet schemes developed by Cohen, Dahmen and DeVore for nonlinear variational problems, we apply this optimal adaptive wavelet scheme to a particular nonlinear partial differential equation studied by Nashed and Scherzer in image denoising. We established the theoretical convergence rates of the adaptive wavelet scheme for such nonlinear variational problem in any dimension. This is joint work with W. Dahmen and V. Pasyuga.