Internal structure of the multiresolution analyses defined by the unitary extension principle

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May 23, 2008

Abstract

We analyze the internal structure of the multiresolution analyses of $L^2(\mathbb{R}^d)$ defined by the unitary extension principle (UEP) of Ron and Shen. Suppose we have a wavelet tight frame defined by the UEP. Define V_0 to be the closed linear span of the shifts of the scaling function and W_0 that of the shifts of the wavelets. Finally, define V_1 to be the dyadic dilation of V_0. We characterize the conditions that $V_1 = W_0$, those that $V_1 = V_0 \dotplus W_0$ and those that $V_1 = V_0 \oplus W_0$. In particular, we show that if we construct a wavelet frame of $L^2(\mathbb{R})$ from the UEP by using two trigonometric filters, then $V_1 = V_0 \dotplus W_0$; and show that $V_1 = W_0$ for the B-spline example of Ron and Shen. A more detailed analysis of the various 'wavelet spaces' defined by the B-spline example of Ron and Shen is also included.

AMS 2000 Subject Classification: 42C15, 42C40.
Key Words: Shift-Invariant Space, Unitary Extension Principle, Multiresolution Analysis.

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