Linearized Bregman Iterations for Compressed Sensing

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Finding a solution of a linear equation $Au = f$ with various minimization properties arises from many applications. One of such applications is compressed sensing, where an efficient and robust-to-noise algorithm to find a minimal ℓ_1 norm solution is needed. This means that the algorithm should be tailored for large scale and completely dense matrices A, while Au and $A^T u$ can be computed by fast transforms and the solution to seek is sparse. Recently, a simple and fast algorithm based on linearized Bregman iteration was proposed in [Osher et.al. 2008, Yin et.al. 2008] for this purpose. In this talk, we give the convergence analysis of linearized Bregman iterations and the minimization properties of their limit. Based on our analysis, we derive also a new algorithm that is proven to be convergent with a rate. Furthermore, the new algorithm is as simple and fast as the algorithm given in [Osher et.al. 2008, Yin et.al. 2008] in approximating a minimal ℓ_1 norm solution of $Au = f$ as shown by numerical simulations. Hence, it can be used as another choice of an efficient tool in compressed sensing.