Stability of Nonlinear Subdivision and Multiscale Transforms

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Abstract

Extending upon previous work by Cohen/Dyn/Matei for (W)ENO schemes, and Kuijt/van Damme, Amat/Liandrat for certain convexity-preserving schemes, we present a general approach to studying Lipschitz stability of nonlinear subdivision schemes and multiscale transforms in the univariate case. Lipschitz stability is a desirable property, e.g., for multiscale compression algorithms. Our main condition for stability is formulated in terms of joint spectral radii for families of nonlinear maps, and turns out to be close to optimal. It covers the special cases (WENO, PPH) considered so far in the literature but also implies the stability in some new cases (median interpolating transform, power-p schemes, etc.). Although the investigation concentrates on multiscale transforms

$$\{v^0, d^1, \dots, d^J\} \quad \longmapsto \quad v^J, \qquad J \ge 1,$$

in $\ell_\infty(\mathbb{Z})$ given by a stationary recursion of the form

$$v^j = Sv^{j-1} + d^j, \qquad j \ge 1,$$

involving a nonlinear subdivision operator S acting on $\ell_\infty(\mathbb{Z})$, the approach is extendable to other nonlinear multiscale transforms, to other norms, and opens an avenue to studying multivariate schemes as well.

The talk is based on joint work with my PhD student S. Harizanov.