Pointwise Properties of Bounded Refinable Function Vectors

Di-Rong Chen

Department of Applied Mathematics
Beijing Univ. of Aeronautics and Astronautics
Beijing 100083, P.R.China

Abstract

A vector $\phi = (\phi_1, \ldots, \phi_r)^T$ of functions is a refinable function vector if it satisfies the refinement equation

$$\phi = \sum_{\alpha \in \mathbb{Z}^s} a(\alpha)\phi(M \cdot - \alpha),$$

where $a = (a(\alpha))_{\alpha \in \mathbb{Z}^s}$ is a finitely supported sequence of $r \times r$ real matrices, and M is an $s \times s$ integer matrix such that $\lim_{n \to \infty} M^{-n} = 0$.

The topic of this talk is the study of pointwise properties concerning L_∞ refinable function vectors. We first discuss the pointwise regularity of refinable function vectors. There are many contributions to the study of global regularity. It is more involved to study the local regularity. Under a mild assumption, we prove that, for $\phi \in L_\infty$, there is a set $H \subseteq \mathbb{R}^s$ of full measure such that $\phi|_H$ has a positive Hölder exponent $\gamma(x)$ at every $x \in H$.

The second issue is the pointwise convergence of cascade algorithm. Starting with an initial vector ϕ_0, a cascade sequence $(\phi_n)_{n=1}^\infty$ is constructed by iteration

$$\phi_n = \sum_{\alpha \in \mathbb{Z}^s} a(\alpha)\phi_{n-1}(M \cdot -\alpha), \qquad n = 1, 2, \ldots.$$

Under the assumption that $(\phi_n)_{n=1}^\infty$ is bounden in L_∞, we show that the a.e. convergence of $(\phi_n)_{n=1}^\infty$ implies that it has a convergence rate of geometry a.e.

The main approaches are the self-affine tile, joint spectral radius and ergodic theorem.