SOLITONS INTERACTIONS OF TWO TRIADS OF THE KADOMTSEV-PETVIASHVILI EQUATION

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Abstract. Soliton solutions of the Kadomtsev-Petviashvili (KP) equation which is a two-dimensional form of the Korteweg-de Vries (KdV) equation could be obtained using Hirota bilinear method. The traditional group-theoretical approach can generate analytical solitons solutions of the KP equation that has infinitely many conservation laws. Two-soliton solutions of the KP equation produce a triad, quadruplet and a non-resonant soliton structures in soliton interactions. From these basic resonant structures, the interactions of higher number of soliton could produce various interactions patterns. This paper focus on the interaction of two triads which is a type of four-soliton solutions of the KP equation. It is shown that the solution obtained is in the form of a Wronskian determinant. Computer simulations using Matlab and C++ of the interactions of four solitons of KP equation were also displayed in this paper.

Keywords. Kadomtsev-Petviashvili, Korteweg de Vries, Hirota bilinear, triad, soliton, resonance and Wronskian.

AMS (MOS) subject classification: 35Q53

1 Introduction

Kadomtsev and Petviashvili [5] derived the Kadomtsev-Petviashvili (KP) equation in 1970 to examine the stability of the one-soliton solution of the Korteweg-de Vries (KdV) equation under transverse perturbations. Miles [6], [7] discovered that the interaction region between the incident solitons and the centered-shifted solitons in two-soliton interaction is essentially itself a single soliton. The interaction soliton is called resonant soliton, which is associated with two incident solitons.

In two-soliton solutions of the KP equation, a few interesting resonant structures were obtained, namely triad, quadruplet and a cross [8], [10], [11]. From these basic resonant structures, the interactions patterns of higher number of solitons of the KP equation could be observed. For example, there are three types of interaction in four-soliton solutions of the KP equation, which are interaction of two triads, interaction of two quadruplets and interaction of a triad and a quadruplet. The interaction of a triad and a quadruplet is expressed in [12]. This paper focus on the interactions of two triads in four solitons solutions of KP equation.
As the number of soliton increases, the process of finding the solution becomes more complicated and tedious. Therefore, this paper discuss about the process of finding the solution by using the Wronskian’s determinant. By using this technique, the process of finding the solutions could be simplified. Some of the computer simulations were displayed in this paper to show the whole process of interactions. This is important as it could be used to study the water waves interactions in the ocean basin.

The outline of this paper is as follows:

The KP equation is described briefly in the next section and the resonant structures in two-soliton solutions were displayed in section three. Section four shows the process of finding the solution of the interactions of two triads by using the Wronskian’s determinant. Computer simulations using Matlab and C++ were used to show the interactions patterns in section five and followed by the conclusion in the last section.

### 2 The Kadomtsev-Petviashvili Equation

The KP equation is given by

\[(u_t + 6 uu_x + u_{xxx})_x = 3 u_{yy}\]  

where the suffixes denote the partial derivatives. Consider the linearized form of Equation (1), then the plane-wave solutions whose phase variable \(kx + my - \omega t\) satisfied the dispersion relationship;

\[\omega = -\left(\frac{3m^2}{k} + k^3\right)\].

Equation (2) can be parameterized using \(k = l + n\) and \(m = n^2 - l^2\), Anker & Freeman,[1] into

\[\omega = \frac{(l + n)^4 + 3(n^2 - l^2)^2}{n + l},\]
\[\omega = -4(l^3 + n^3).\]  

\(N\)-soliton solution for the KP equation had been solved by Satsuma [9] using Hirota bilinear method [4] and is given by

\[u(x, y, t) = 2 \frac{\partial^2}{\partial x^2} (\ln f),\]
\[= 2 \left[ f_{xx} f - f_x^2 \right]\]  

with the function \(f(x, y, t)\) given by

\[f(x, y, t) = \left[ \delta_{ij} + \frac{\alpha_i}{l_i + n_j} \exp(\eta_i) \right]\]
where $\delta_{ij}$ is Kronecker delta, $\eta_i = k_ix + m_iy - \omega_i t$ and $a_i$ is the arbitrary constant.

3 Two-Soliton Solutions of the KP Equation

We shall look into the two solitons solutions of the KP equation first with $N=2$, then the function $f(x,y,t)$ is given as

$$f(x,y,t) = 1 + \varepsilon_1 \exp(\eta_1) + \varepsilon_2 \exp(\eta_2) + A_{12} \varepsilon_1 \varepsilon_2 \exp(\eta_1 + \eta_2) \quad (7)$$

where

$$A_{12} = \frac{(n_1-n_2)(l_1-l_2)}{(l_1+n_2)(l_2+n_1)} \quad (8)$$

and

$$\varepsilon_i = \frac{a_i}{l_i + n_i}. \quad (9)$$

The value of $A_{12}$ plays an important role in determining the type of resonance occurrence. If $A_{12} = 0$, then it is a full resonance and the resonant structure is called a triad, $T_{12}$ as given in Figure 1. In order to have partial resonance or quadruplet, $Q_{12}$ the value of $A_{12}$ must be very close to zero as given in Figure 3. If $A_{12} \neq 0$, then the resonant structure is known as a cross, $C_{12}$ and it is said to be non-resonance as given in Figure 5. All these resonant structures and its contour plot are shown from Figure 1 to Figure 6.

From Figure 1 and 2, soliton 1, $S_1$ interacts with soliton 2, $S_2$ and produced a resonant soliton $S_{12}$. This structure is called a triad. On the other hand, the resonant soliton breaks into $S_1$ and $S_2$ again after interaction in partial resonance case as shown in Figure 3 and 4. The resonant structure is called a quadruplet. However, $S_1$ and $S_2$ do not interact among themselves in non-resonance case. They simply cross each other without interaction. This is shown in Figure 5 and 6.

![Figure 1: A triad, $T_{12}$.](image-url)
Figure 2: Contour plot of a triad.

Figure 3: A quadruplet, Q_{12}.

Figure 4: Contour plot of a quadruplet.
4 Solution of the Interaction of Two Triads

In this section, we shall consider the interactions of four solitons of KP equation and the function $f(x, y, t)$ is given by

$$f(x, y, t) = \delta_{ij} + \frac{a_i}{\delta_{ij}} \exp(\theta_i + \gamma_j)$$

(10)

where

$$\theta_i = \lambda_i x - \lambda_i^2 y + 4\lambda_i^3 t,$$

(11)

$$\gamma_j = \kappa_j x + \kappa_j^2 y + 4\kappa_j^3 t, \quad i, j = 1, 2, 3 \text{ and } 4.$$  

(12)

Freeman and Nimmo [2] had showed that the determinant in Equation (10) is a Wronskian determinant. Equation (10) can be rewritten as

$$f = |I + X|$$

(13)
where $I$ is an identity matrix and $X$ is given as

$$
X = \begin{bmatrix}
X_{11} & X_{12} & X_{13} & X_{14} \\
X_{21} & X_{22} & X_{23} & X_{24} \\
X_{31} & X_{32} & X_{33} & X_{34} \\
X_{41} & X_{42} & X_{43} & X_{44}
\end{bmatrix} \quad (14)
$$

with $X_{ij} = a_i e^{\theta_i + \gamma_j}$.

In this paper, we will discuss about the interactions of two triads. Thus, soliton 1, $S_1$, is chosen to interact with soliton 3, $S_3$, to produce a triad, $T_{13}$, while soliton 2, $S_2$, interacts with soliton 4, $S_4$, to produce another triad, $T_{24}$. In order to produce these triads, a condition where $n_1 = n_3$, $n_2 = n_4$ and $n_1 = n_3 \approx n_2 = n_4$ is needed so that $T_{13}$ will interact with $T_{24}$. In other words, $\gamma_1 = \gamma_3$ and $\gamma_2 = \gamma_4$. Hence, the function $f(x, y, t)$ becomes

$$
f = \begin{bmatrix}
1 + X_{11} & X_{12} & X_{11} & X_{12} \\
X_{21} & 1 + X_{22} & X_{21} & X_{22} \\
X_{31} & X_{32} & 1 + X_{31} & X_{32} \\
X_{41} & X_{42} & X_{41} & 1 + X_{42}
\end{bmatrix} \quad (15)
$$

After performing matrix row operations and simplifications, the function $f(x, y, t)$ becomes

$$
f = \begin{bmatrix}
1 + X_{11} + X_{31} & X_{12} + X_{32} \\
X_{21} + X_{41} & 1 + X_{22} + X_{42}
\end{bmatrix}. \quad (16)
$$

By using the matrix transposition, Equation (16) is rewritten as

$$
f = \begin{bmatrix}
1 + X_{11} + X_{31} & X_{21} + X_{41} \\
X_{12} + X_{32} & 1 + X_{22} + X_{42}
\end{bmatrix}. \quad (17)$$
By substituting $X_{ij}$ back into Equation (17), we obtain

$$f = \begin{vmatrix}
1 + \frac{a_1 e^{\theta_1 + \gamma_1}}{l_1 + n_1} & \frac{a_2 e^{\theta_2 + \gamma_1}}{l_2 + n_1} & \frac{a_4 e^{\theta_4 + \gamma_1}}{l_4 + n_1} \\
\frac{a_1 e^{\theta_1 + \gamma_2}}{l_1 + n_2} & \frac{a_3 e^{\theta_3 + \gamma_2}}{l_3 + n_2} & 1 + \frac{a_2 e^{\theta_2 + \gamma_2}}{l_2 + n_2} + \frac{a_4 e^{\theta_4 + \gamma_2}}{l_4 + n_2}
\end{vmatrix}. \quad (18)$$

Taking out the factor $e^{\gamma_1}$ and $e^{\gamma_2}$, we obtain

$$f = e^{\gamma_1} e^{\gamma_2} \begin{vmatrix}
e^{-\gamma_1} + \frac{a_1 e^{\theta_1}}{l_1 + n_1} & \frac{a_3 e^{\theta_3}}{l_3 + n_1} & \frac{a_2 e^{\theta_2}}{l_2 + n_1} + \frac{a_4 e^{\theta_4}}{l_4 + n_1} \\
\frac{a_1 e^{\theta_1}}{l_1 + n_2} & \frac{a_3 e^{\theta_3}}{l_3 + n_2} & e^{-\gamma_2} + \frac{a_2 e^{\theta_2}}{l_2 + n_2} + \frac{a_4 e^{\theta_4}}{l_4 + n_2}
\end{vmatrix}. \quad (19)$$

From Equation (12), it could be noticed that the exponential term $e^{\gamma_1}$ and $e^{\gamma_2}$ is linear in $x$. Therefore it will becomes zero after the second derivaties with respect to $x$ when we substitute into Equation (5). Thus the term $e^{\gamma_1}$ and $e^{\gamma_2}$ can be omitted and Equation (19) becomes,

$$f = e^{\gamma_1} e^{\gamma_2} \begin{vmatrix}
e^{-\gamma_1} + \frac{a_1 e^{\theta_1}}{l_1 + n_1} & \frac{a_3 e^{\theta_3}}{l_3 + n_1} & \frac{a_2 e^{\theta_2}}{l_2 + n_1} + \frac{a_4 e^{\theta_4}}{l_4 + n_1} \\
\frac{a_1 e^{\theta_1}}{l_1 + n_2} & \frac{a_3 e^{\theta_3}}{l_3 + n_2} & e^{-\gamma_2} + \frac{a_2 e^{\theta_2}}{l_2 + n_2} + \frac{a_4 e^{\theta_4}}{l_4 + n_2}
\end{vmatrix}. \quad (20)$$

The above matrix could be written as the sum of three matrices.

$$f = \left| E_N + E_{L_1} + E_{L_2} \right|, \quad (21)$$

where

$$E_N = \begin{bmatrix}
e^{-\gamma_1} & 0 \\
0 & e^{-\gamma_2}
\end{bmatrix}, \quad (22)$$

$$E_{L_1} = \begin{bmatrix}
\frac{a_1 e^{\theta_1}}{l_1 + n_1} & \frac{a_2 e^{\theta_2}}{l_2 + n_1} \\
\frac{a_1 e^{\theta_1}}{l_1 + n_2} & \frac{a_2 e^{\theta_2}}{l_2 + n_2}
\end{bmatrix}, \quad (23)$$

and
E_{L_2} = \begin{bmatrix} \frac{a_3 e^{\theta_3}}{l_3 + n_1} & \frac{a_4 e^{\theta_4}}{l_4 + n_1} \\ \frac{a_3 e^{\theta_3}}{l_3 + n_2} & \frac{a_4 e^{\theta_4}}{l_4 + n_2} \end{bmatrix}.

(24)

Rewrite Equation (23) and Equation (24) as follow:

\begin{align*}
E_{L_1} &= \begin{bmatrix} \frac{1}{l_1 + n_1} & \frac{1}{l_2 + n_1} \\ \frac{1}{l_1 + n_2} & \frac{1}{l_2 + n_2} \end{bmatrix} \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} e^{\theta_1} & 0 \\ 0 & e^{\theta_2} \end{bmatrix}, \\
E_{L_2} &= \begin{bmatrix} \frac{1}{l_3 + n_1} & \frac{1}{l_4 + n_1} \\ \frac{1}{l_3 + n_2} & \frac{1}{l_4 + n_2} \end{bmatrix} \begin{bmatrix} a_3 & 0 \\ 0 & a_4 \end{bmatrix} \begin{bmatrix} e^{\theta_3} & 0 \\ 0 & e^{\theta_4} \end{bmatrix},
\end{align*}

(25)

From Equation (25) and Equation (26),

\begin{equation}
\begin{align*}
f &= |E_N + M_1 A_1 E_{l_1} + M_2 A_2 E_{l_2}|.
\end{align*}
\end{equation}

(27)

The matrix $M = \begin{bmatrix} 1/l_a + n_b \end{bmatrix}$ is related to the Van der Monde matrices $V = [(-n_b)^{i-1}]$ and $W = [(-1)^{i-1}(l_a)^{i-1}]$, Freeman,[3]. By introducing the two diagonal matrices

\begin{equation}
P = [\delta_{ij} \prod_{p, p \neq b} (n_p - n_b)] \quad \text{and} \quad Q = [\delta_{ij} (-1)^{i-1} \prod_{a} (l_a + n_b)],
\end{equation}

(28)

we have

\begin{equation}
V^{-1}W = P^{-1}MQ
\end{equation}

(29)

so that

\begin{equation}
M = PV^{-1}WQ^{-1}.
\end{equation}

(30)

Thus we have

\begin{equation}
\begin{align*}
f &= |E_N + PV^{-1}W_1 Q_1^{-1} A_1 E_{l_1} + PV^{-1}W_2 Q_2^{-1} A_2 E_{l_2}|, \\
&= |PV^{-1}| |V^{-1}E_N + W_1 Q_1^{-1} A_1 E_{l_1} + W_2 Q_2^{-1} A_2 E_{l_2}|.
\end{align*}
\end{equation}

(31)
where

\[
V = \begin{bmatrix} 1 & 1 \\ -n_1 & -n_2 \end{bmatrix}, \quad P = \begin{bmatrix} n_2 - n_1 & 0 \\ 0 & n_1 - n_2 \end{bmatrix},
\]

\[
W_1 = \begin{bmatrix} 1 & -1 \\ l_1 & -l_2 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 & -1 \\ l_3 & -l_4 \end{bmatrix},
\]

\[
Q_1 = \begin{bmatrix} (l_1 + n_1)(l_1 + n_2) & 0 \\ 0 & -(l_2 + n_1)(l_2 + n_2) \end{bmatrix},
\]

\[
Q_2 = \begin{bmatrix} (l_3 + n_1)(l_3 + n_2) & 0 \\ 0 & -(l_4 + n_1)(l_4 + n_2) \end{bmatrix}.
\]

Since \(PV^{-1}\) is a linear function in \(x\), thus it could again be omitted. Hence

\[
f = \left|VP^{-1}E_N + W_1Q_1^{-1}A_1E_{l_1} + W_2Q_2^{-1}A_2E_{l_2}\right|
\]

where

\[
VP^{-1}E_N = \begin{bmatrix} e^{-\gamma_1} & e^{-\gamma_2} \\ -\frac{n_1 e^{-\gamma_1}}{n_1 - n_2} & -\frac{n_2 e^{-\gamma_2}}{n_1 - n_2} \end{bmatrix},
\]

\[
W_1Q_1^{-1}A_1E_{l_1} = \begin{bmatrix} \frac{a_1 e^{\beta_1}}{(l_1 + n_1)(l_1 + n_2)} & \frac{a_2 e^{\beta_2}}{(l_2 + n_1)(l_2 + n_2)} \\ \frac{l_1 a_1 e^{\beta_1}}{(l_1 + n_1)(l_1 + n_2)} & \frac{l_2 a_2 e^{\beta_2}}{(l_2 + n_1)(l_2 + n_2)} \end{bmatrix},
\]

\[
W_2Q_2^{-1}A_2E_{l_2} = \begin{bmatrix} \frac{a_3 e^{\beta_3}}{(l_3 + n_1)(l_3 + n_2)} & \frac{a_4 e^{\beta_4}}{(l_4 + n_1)(l_4 + n_2)} \\ \frac{l_3 a_3 e^{\beta_3}}{(l_3 + n_1)(l_3 + n_2)} & \frac{l_4 a_4 e^{\beta_4}}{(l_4 + n_1)(l_4 + n_2)} \end{bmatrix}.
\]
Substituting Equation (37), (38) and (39) into Equation (36) transformed the function \( f(x, y, t) \) into a Wronskian determinant which is in the form of

\[
f = \begin{vmatrix} \phi_1 & \phi_2 \\ \frac{\partial \phi_1}{\partial x} & \frac{\partial \phi_2}{\partial x} \end{vmatrix},
\]

(40)

where

\[
\phi_1 = -e^{-\gamma_1} + \frac{a_1\epsilon_1}{n_1 + n_2} + \frac{a_3\epsilon_3}{l_3 + n_1(l_3 + n_2)},
\]

(41)

and

\[
\phi_2 = e^{-\gamma_2} + \frac{a_2\epsilon_2}{n_1 - n_2} + \frac{a_4\epsilon_4}{(l_4 + n_1)(l_4 + n_2)}.
\]

(42)

Solving Equation (40) yields

\[
f = 1 + \epsilon_1e^{\eta_1} + \epsilon_2e^{\eta_2} + \epsilon_3e^{\eta_3} + \epsilon_4e^{\eta_4} + A_{12}\epsilon_1\epsilon_2e^{\eta_1-\eta_2} + A_{14}\epsilon_1\epsilon_4e^{\eta_1+\eta_4} + A_{23}\epsilon_2\epsilon_3e^{\eta_2+\eta_3} + A_{34}\epsilon_3\epsilon_4e^{\eta_3-\eta_4},
\]

(43)

which is the \( f(x, y, t) \) function for two triads in the four solitons solutions of KP equation where,

\[
A_{ij} = \frac{(l_i - l_j)(n_i - n_j)}{(l_i + n_i)(l_i + n_j)} \quad \text{and} \quad \epsilon_i = \frac{a_i}{l_i + n_i}.
\]

5 Computer Simulations of Interactions Patterns of Two Triads

The following computer simulations of the interactions patterns of two triads were shown in this section. In order to have two triads interacting, the values of \( n_i, l_i \) with \( i = 1, 2, 3, 4 \) were given in Table 1. The values of \( n_1 \) and \( n_4 \) are equal in order to produce a triad, \( T_{13} \) while \( n_2 = n_4 \) to produce another triad, \( T_{24} \). To let both triads interacts among themselves, the values of \( n_1 \) and \( n_3 \) are chosen to be very close to \( n_2 \) and \( n_4 \). Otherwise, both triads would not interact.

<table>
<thead>
<tr>
<th>( n_1 = 3 )</th>
<th>( n_2 = 3 + 10^{-15} )</th>
<th>( n_3 = 3 )</th>
<th>( n_4 = 3 + 10^{-15} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1 = 1 )</td>
<td>( l_2 = 2 )</td>
<td>( l_3 = -1 )</td>
<td>( l_4 = 3 )</td>
</tr>
</tbody>
</table>

Table 1: The values of \( n_i, l_i, i = 1, 2, 3, 4 \)

Every soliton is centered along the line \( kx + my = 0 \) at \( t = 0 \). Thus, from the values in Table 1, each soliton is centered along the respective lines as shown in Table 2.
Solitons Interactions of Two Triads

<table>
<thead>
<tr>
<th>Soliton</th>
<th>S_1</th>
<th>y = -\frac{1}{2}x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soliton 2</td>
<td>S_2</td>
<td>y \approx -x</td>
</tr>
<tr>
<td>Soliton 3</td>
<td>S_3</td>
<td>y = -\frac{1}{3}x</td>
</tr>
<tr>
<td>Soliton 4</td>
<td>S_4</td>
<td>x \approx 0</td>
</tr>
</tbody>
</table>

Table 2: The geometrical line representation of solitons

Therefore, the position of each soliton can be easily determined in the simulation. The interaction are shown from Figure 7 to Figure 15.

From Figure 7, it is obvious that there are two triads, T_{13}, T_{24} and a quadruplet, Q_{34} at time t = 0.5. T_{13} is produced by S_1 and S_3 while T_{24} is produced by S_2 and S_4. Due to the values of n_3 and n_4, S_3 interacts with S_4 to produce Q_{34}. Figure 8 shows the contour plot of the interaction of two triad at time, t = 0.5 and the lines were represented geometrically as in Table 2.

Both triads move towards each other as shown in Figure 9 and 10. They start to interact with Q_{34} at time t = 0 as shown in Figure 11 and 12. The interaction produces two parallelogram shapes which could be observed clearly at the end of the interaction (refer to Figure 13 and 14). The size of these parallelogram shapes increases as time changes. This is shown in Figure 15.

Figure 7: Interaction of two triad at t = 0.5

Figure 8: Contour plot of the interaction of two triad at t = 0.5
Figure 9: Interaction of two triad at $t = 0.25$

Figure 10: Contour plot of the interaction of two triad at $t = 0.25$

Figure 11: Interaction of two triad at $t = 0$
Figure 12: Contour plot of the interaction of two triad at $t = 0$

Figure 13: Interaction of two triad at $t = -0.25$

Figure 14: Contour plot of the interaction of two triad at $t = -0.25$
These interaction patterns can be represented geometrically. From Equation (43), we have

\[
f(x, y, t) = \sum_{i=1}^{5} \varepsilon_i e^{\alpha_i t} + \sum_{j=6}^{9} A_{ij} \varepsilon_i e^{\alpha_i t} + \sum_{k=10}^{12} B_{ik} \varepsilon_i e^{\alpha_i t} + \sum_{m=13}^{15} C_{im} \varepsilon_i e^{\alpha_i t} + A_{12} \varepsilon_1 \varepsilon_2 e^{\alpha_1 t} + A_{14} \varepsilon_1 \varepsilon_4 e^{\alpha_1 t} + A_{23} \varepsilon_2 \varepsilon_3 e^{\alpha_2 t} + A_{34} \varepsilon_3 \varepsilon_4 e^{\alpha_3 t} (44)
\]

Any combination of two terms of the function \( f(x, y, t) \) will give a soliton solution. Thus, for each soliton, the combinations are as follows:

| Soliton 1, \( S_1 \) | (12), (36), (57), (45) |
| Soliton 2, \( S_2 \) | (13), (26), (48), (46) |
| Soliton 3, \( S_3 \) | (14), (38), (59), (47) |
| Soliton 4, \( S_4 \) | (15), (27), (49), (48) |
| Resonant Soliton, \( S_{12} \) | (23), (16), (49) |
| Resonant Soliton, \( S_{13} \) | (24), (68), (79), (50) |
| Resonant Soliton, \( S_{24} \) | (35), (67), (89), (51) |
| Resonant Soliton, \( S_{34} \) | (45), (19), (52) |

The geometrical representation of interaction of two triads at \( t = 0.5 \) is shown in Figure 16 while Figure 17 shows the geometrical representation of interaction of two triads at \( t = -0.5 \). From Figure 16 and 17, every soliton can be represented or labeled using the combination of two terms in Equation (16) as listed from Equation (45) to (52).
6 Conclusion

This paper has shown that the solution of the interaction of two triads of the KP equation is actually a Wronskian’s determinant. In other words, the solution of the interactions of two triads could be solved by using the Wronskian’s determinant. This is important as it can simplify the computations process when the number of soliton increases.

Computer simulations of the interaction were shown in this paper and it shows that this interactions will generate interesting patterns of two triads of KP equation. In future, the interactions of higher number of soliton could be
generated and we can solve higher order solitons solutions of KP equation. With this computational abilities, we can now see the water waves interactions patterns in the ocean. We may be able to predict what is happening the the rough ocean basin in future by computer simulations.

7 References


