IMS School Lecture

Reviving the Language Used to Invent the Calculus

by Prof Peter Loeb (University of Illinois)

We will discuss the construction and use of infinitely large and infinitely small numbers. This will include the role these numbers played in the invention of calculus, their fall from mathematical grace in the early twentieth century, and their recent rehabilitation.

For a positive number to be infinitesimal, it must be greater than 0 and yet smaller than any positive number you might write using a decimal expansion. No matter how many zeros you might start with in that expansion, a positive infinitesimal number will be still smaller.

In spite of the mysteriousness of this idea, infinitesimal numbers have been used in mathematics for more than 2000 years. After the work of the Greek mathematicians, they were again introduced in the development of the calculus, which began in the late 1600's with the work of Newton and Leibniz. They were always regarded as a useful fiction that facilitated mathematical computation and invention. In the late 1940's, mathematicians began removing infinitesimals from all mathematics courses. Then, in about 1960, when the removal of infinitesimals from rigorous mathematics was complete, Abraham Robinson established a rigorous foundation for the use of infinitesimals in all fields of mathematics.

Robinson's work started a vigorous field of research with applications in many areas. For example, there are many results, some used in applications for decades, without a rigorous foundation, that are only true when based on probability spaces constructed from his models.

About the Speaker

Peter Loeb, a professor of mathematics at the University of Illinois at Urbana-Champaign, received his Ph.D. degree under H. L. Royden at Stanford University in 1964. His 75 publications deal with real analysis, including Robinson's infinitesimal analysis. An important contribution (the subject of his 1983 talk at the International Congress of Mathematicians in Warsaw) was the construction of measure spaces that allows infinite stochastic processes to be treated with the combinatorial tools available for finite processes. That invention of "Loeb spaces" has become the most influential tool for applying methods of mathematical logic to real analysis and probability. He has taught numerous introductory calculus courses from both the nonstandard and the standard viewpoint, endeavoring in the latter setting to retain the ease of the former. His other interests in electronics, photography and computers are a natural outgrowth of his belief that he who dies with the most toys wins.

Venue: Victoria Junior College LT 1, Date: 19 May 2005, Thursday, Time: 2.30-3.30pm

Jointly organized by

Institute for Mathematical Sciences, NUS

Victoria Junior College