Financial Contracts and Enforcement

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Outline of Part 2: Enforcement

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1 Introduction

The study of contract enforcement is important for at least two reasons. First, contract enforcement is intrinsically important because people may subsequently deviate from initial promises in a dynamic setting when the environment can change (e.g., information is revealed) unless they have an incentive to honor the initial commitments. However, in economics enforcement has often been assumed rather than modelled explicitly. It is common to explicitly or implicitly assume there is some ex post enforcement authority that costlessly commits people to their initial decisions. Second, in the Costly State Verification (CSV) model we saw that debt was ex post inefficient. This means that if agents could, they would like to recontract ex post. In this section we will see that altering the "commitment friction" can have interesting effects on financial contracts. We will consider the Krasa and Villamil [8] Costly Enforcement model which treats enforcement as an explicit decision variable that is chosen optimally.

There are four main approaches to enforcement:

1. Perfect Ex Post Enforcement: Early CSV models (e.g., Townsend [12] or Gale and Hellwig [5]). These models assume perfect ex post enforcement by an exogenous "court."

2. Exclusion: Sovereign Debt models (Eaton and Gersovitz [4]). These models assume no enforcement is possible, thus contracts must be self-enforcing. The implicit punishment is exclusion (i.e., autarky).

3. Willful default and Costly Enforcement: Krasa and Villamil [8] and Krasa, Sharma and Villamil [7] model costly enforcement as an explicit decision variable.

4. Renegotiation: When there is limited commitment to initial agreements, agents may wish to "settle out of court:" Krasa, Sharma and Villamil [6]

In the Costly Enforcement model, Krasa and Villamil [8] study contracts with limited commitment. They define stochastic or deterministic contracts as an equilibrium property of a dynamic game with incomplete information. They show that:

- When commitment is limited, deterministic contracts are optimal (SDCs).

- When there is full commitment to the initial proposal, stochastic contracts dominate deterministic contracts.

When commitment is limited, agents may have an incentive to renegotiate the initial contract after the borrower observes state y. Further, any renegotiation payment by the borrower provides information about the state.[1] KV make two key modifications to the CSV structure:

1. Commitment to the initial contract may be limited.
2. The lender decides whether to enforce based on an observed "voluntary" payment by the borrower.

2 Costly Enforcement Model

Consider an economy with two risk neutral agents, a planning period, and three subsequent periods. Agents derive utility only from consumption in the final period. The investor has one unit of a consumption/investment good in the initial period and no endowment in any other period. The penniless entrepreneur owns a production technology which is described by a random variable with finitely many realizations $y \in Y = \{\underline{y}, ..., \bar{y}\}$. The technology transforms one unit of the input into y units of output. Agents share a common prior belief $\beta(y) > 0$ about the possible realizations, and know that the entrepreneur will privately observe output.

Because the entrepreneur has a technology but no input and the investor has an input but no technology, production occurs only if the investor can be persuaded to transfer the good to the entrepreneur. This is done by writing a *contract* which consists of payments by the entrepreneur in subsequent periods. In the first period nature chooses the outcome of the venture. In the second period the entrepreneur decides whether to make a voluntary payment chosen from a subset of project returns that includes the possibility of making no payment. In the last period the investor, after having observed the entrepreneur's payment but not the state, decides whether to enforce a final payment. Enforcement is provided by a costly technology called a court.

In the contracting literature (e.g., CSV model), costless ex-post enforcement of the payment is assumed. The Costly Enforcement model differs from the literature in two key respects. First, the entrepreneur decides whether to honor an unenforceable initial payment promise υ. If υ is zero no payment occurs, but if it is positive the transfer is made immediately. Second, we introduce a legally enforceable payment $\ell(\cdot)$. The investor has the ability to compel payment of ℓ by choosing an

[1] This can lead to multiple ex-post equilibria.

action e, after the voluntary payment action has been observed. If $e = 1$ then ℓ is enforced and if $e = 0$ it is not. Thus a contract in our model is a pair of payments with associated probabilities for choosing whether to make the voluntary payment or enforce the legal payment.

The Costly Enforcement model is a stylized version of the enforcement provided by courts. The assets available before enforcement are $y - v$, where y is the entrepreneur's output realization and v is the voluntary payment. The court's role, if enforcement is requested, is to determine the funds available after any voluntary payment, and then enforce payment ℓ which is contingent on these funds. The model assumes that the court's enforcement technology is imperfect: the court cannot seize an amount $\bar{x}$.[2] The amount available for transfer is then $x = \max\{y - v - \bar{x}, 0\}$, where no funds can be transferred if the amount that can be hidden $\bar{x}$ exceeds $y - v$. Assume that enforcement is requested by the investor. When it occurs the investor pays a positive cost c_I and the entrepreneur pays a positive cost c_E. These costs are a deadweight loss, thus both agents have an incentive to minimize the court's use.

Payments v and ℓ define a noncooperative game with incomplete information with associated strategies $\mathscr{v}(y)$ and $\mathscr{e}(v)$. Strategy $\mathscr{v}(y)$ is the probability the entrepreneur assigns to a particular voluntary payment v and $\mathscr{e}(v)$ is the probability that the investor chooses to enforce payment ℓ. The strategies are used to choose v and e optimally as part of a perfect Bayesian Nash equilibrium (PBNE). The optimal contract maximizes agents' expected utilities subject to resource and time consistency constraints, and a constraint that the strategies are a PBNE.

Characterizing the form of the optimal contract amounts to asking the question– do agents choose deterministic or random (behavioral) strategies? The game that underlies the contract problem permits random strategies because *whether contracts are stochastic or deterministic is a property of the equilibrium of the game.* A given contract defines a game that is summarized by a set of players, strategies, a production technology, beliefs, and payoffs. Initial beliefs are given by a prior $\beta(y)$ over the outcomes. Figure 1 indicates that nature moves first and chooses $y \in Y$ at time 1, which the entrepreneur privately observes. The game involves two actions: which voluntary payment v to make at time 2 and whether to go to court to enforce ℓ at time 3 (yes if $e = 1$, and no if $e = 0$). The entrepreneur uses strategy $\mathscr{v}(y)$ to choose v, and the investor uses strategy $\mathscr{e}(v)$ to choose e. The investor uses publicly observable action v to update beliefs to $\beta(y|v)$.

[2]This can occur, for example, if the entrepreneur can abscond with or hide an amount $\bar{x}$.

Instead of a pair (R, B) in the CSV model, the objects of interest in the CE model are $(\upsilon, \ell, \mathscr{v}(y), \not{e}(\upsilon), \beta(y|\upsilon))$. Payment $\upsilon \geq 0$ is a payment made by the entrepreneur to the investor, if it is optimal for the entrepreneur to do so. Payment υ cannot be compelled and it cannot be retracted once made; it is "money on the table." The investment problem is a multi-stage game with imperfect information because beliefs are allowed to vary endogenously as information changes during the game. Formally,

$\mathscr{v}(y)$: an entrepreneur strategy to select voluntary payment υ which assigns a probability to each payment $\upsilon \in V$ given realization $y \in Y$.[3]

$\ell(\cdot, \upsilon)$: a legally enforceable payment function.

$\not{e}(\upsilon)$: a lender enforcement strategy which assigns a probability to the enforcement choice given observed payment υ, where enforcement is a binary decision variable $e = \{0, 1\}$. If $\not{e}(\upsilon) = 1$ the lender requests enforcement of $\ell(x, \upsilon)$ and if $\not{e}(\upsilon) = 0$ the lender does not request enforcement).

$\beta(y|\upsilon)$: the lender's updated belief about the return at $t = 2$, which is derived using Bayes rule from the agents' common prior β over $[\underline{y}, \bar{y}]$ at $t = 0$ (i.e., the probability density function $f(y)$.

The figure below illustrates the four time periods:

t=0: The contract is signed.

t=1: The state y is realized.

t=2: The entrepreneur chooses "voluntary" payment υ, and the lender updates belief β about y after observing υ.

t=3: The lender chooses whether to enforce payment $\ell(\cdot)$.

Figure 0: The Game

		y	υ	e
	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Belief:		$\beta(y)$	$\beta(y\|\upsilon)$	
Strategy:			$\mathscr{v}(y)$	$\not{e}(\upsilon)$

[3] V is a countable set. Krasa and Villamil [8] prove that the optimal payment reduce to only two values, 0 and υ.

Let c_E and c_I be the agents' fixed costs of enforcement, $\bar{x}$ be an amount the borrower can hide from the court, and $x = \max\{y - v - \bar{x}, 0\}$. The agents' ex post payoffs are:

$$\Pi_E(y, v, e) = y - v - e[\ell(x, v) + c_E]$$
$$\Pi_I(y, v, e) = v + e[\ell(x, v) - c_I]$$

The solution concept is the *Perfect Bayesian Nash Equilibrium (PBNE).* As is standard, a PBNE in an extensive form game (i) requires the strategy profiles ($\tilde{v}$ and $\tilde{e}$) to be sequentially rational given belief system β, and (ii) the beliefs to be derived from the strategy profile through Bayes rule whenever possible. Thus, beliefs are formally part of the equilibrium (cf., Mas-Colell, Whinston and Green).

Definition 1 *Strategies $\tilde{v}(y)$, $\tilde{e}(v)$ and beliefs $\beta(y)$, $\beta(y|v)$ are a PBNE* if and only if

(i) $\tilde{v}(y)$ maximizes $E_{\tilde{v}(y),\tilde{e}(v)}\Pi_E(y, v, e)$ for every y.
(ii) $\tilde{e}(v)$ maximizes $\sum_{y\in Y} \beta(y|v)E_{\tilde{e}(v)}\Pi_I(y, v, e)$ for every v.
(iii) $\beta(y|v)$ is derived using Bayes' rule whenever possible.

Conditions (i) and (ii) require each strategy to be a sub-game perfect Nash equilibrium given beliefs. Condition (iii) describes how the investor forms beliefs after observing the borrower's voluntary payment action.

We now state the contract problem.

Problem KV. At $t = 0$ choose $\tilde{v}(y)$, $\tilde{e}(v)$, $\ell(y, v)$ to maximize

$$\sum_y \beta(y)E_{\tilde{v}(y),\tilde{e}(v)}\Pi_I(y, v, e)$$

subject to:

(1.1) $\sum_y \beta(y)E_{\tilde{v}(y),\tilde{e}(v)}\Pi_E(y, v, e) \geq \bar{u}_E$

(1.2) $0 \leq v \leq y$ and $0 \leq \ell(\cdot) \leq x$ for all y, v

(1.3) $\tilde{v}(y)$, $\tilde{e}(v)$, $\beta(y)$, $\beta(y|v)$ is a PBNE at $t = 1$

(1.4) v, $\ell(\cdot)$, $\tilde{e}(v)$ is time consistent.

In Problem KV, agents choose a contract at $t = 0$ to maximize the investor's expected utility subject to:

(1.1) is individual rationality: entrepreneur expected utility is at least $\bar{u}_E$ in every state.

(1.2) is payment feasibility for all y, v.[4]

(1.3) is the PBNE restriction in lieu of incentive compatibility.

(1.4) is time consistency (which ex post efficiency requires).

In Krasa and Villamil [8], the problem included the set of voluntary payments V as a choice variable. Because they prove that only two payments, 0 or $\bar{v}$,[5] are optimal, attention can be restricted to strategy $v(y)$ where $v > 0$ indicates the firm voluntarily pays v and 0 indicates no payments is made. The proof considers two states y, y' in which no enforcement occurs. Total payments to the lender are $v(y) = v$ and $v(y') = v'$. If $v > v'$, the entrepreneur would always choose the lower payment. Hence, all non-bankruptcy payments are the same and equal to $\bar{v}$. Now consider two states y, y' for which enforcement occurs. Total payments are $T = v + \ell(y, v)$ and $T' = v' + \ell(y', v')$. Define an alternative contract with $\ell(y, 0) = T$ and $\ell(y', 0) = T'$; the total payment is the same if the entrepreneur chooses zero repayment at $t = 1$. This alternative contract fulfills all constraints of Problem KV.

Agents have the opportunity to alter the $t = 0$ contract at $t = 2$, but (1.4) ensures they will again choose the original plan.[6] (1.4) states

- Agents cannot recontract in the future to increase expected payoffs.
- All possibilities to alter the initial contract are foreseen at $t = 0$. This constrains the choice of the initial contract.

The opportunity to alter the $t = 0$ contract occurs after the voluntary payment action, but before the enforcement action is chosen. The timing of v is important because it implies limited commitment. The voluntary payment is "money on the

[4]Non-negativity prohibits the borrower from extorting further payments from the lender.

[5]We will often write payment $\bar{v}$ simply as v.

[6](1.4) is similar to (4) in Dewatripont [3], p. 599. Time consistency means that when agents have the opportunity to choose an alternative contract at $t = 2$, $(v', \ell', e(v)')$, the continuation contract is the same as the initial contract. By modelling these future opportunities via (1.4), Krasa and Villamil [8] use a cooperative approach rather than specifying a non-cooperative game. The goal of both approaches is to find Pareto superior allocations. The absence of Pareto improvements precludes renegotiation from actually occurring. Sharma [11] and Krasa, Sharma and Villamil [6] study some interesting tradeoffs that can arise when the time consistency constraint binds. In the last Lecture Note we will study renegotiation in more detail.

table" that cannot be retracted by the borrower or court once it has been made. It provides two opportunities:

1. The voluntary payment can be increased to $v' \geq v$ if both agents agree.
2. Agents can change the enforceable payment to $\ell'(\cdot)$.

The total payment ignoring enforcement costs is $v' + e\ell'(\cdot)$.

Point. By paying $v' > v$, the entrepreneur can induce the investor to refrain from enforcement or to enforce with lower probability. This saves deadweight enforcement costs. The surplus can be used to make a Pareto improvement (i.e., higher payoffs net of enforcement costs).

Theorem 1 gives conditions under which SDC are optimal in the Costly Enforcement model even when stochastic monitoring is possible. Assumptions A.1 and A.2 are necessary for the result:

A.1: $0 < \bar{x} - c_E < \underline{y}$.
A.2: $\underline{y} < \sum_{y<y_k}(y - c)\beta(\{y|y < y_k\})$, where $c = c_I + c_E$.

A.1 and A.2 are conditions on the parameters that determine entrepreneur and investor minimal payoffs from enforcement, respectively. Recall that $\bar{x}$ is the amount of funds that the entrepreneur can hide, c_E is a deadweight enforcement cost paid by the entrepreneur, and $\underline{y}$ is the lowest output realization. A.1 indicates that when the court seizes funds the entrepreneur's payoff after enforcement, net of costs, is small but positive. Most enforcement technologies are likely to have some degree of imperfection. Theorem 1 shows that even when this imperfection is very small and commitment is limited, simple debt is the optimal contract.[7] A.2 indicates that the investor's expected payoff from a simple debt contract when bankruptcy occurs is larger than $\underline{y}$, the payment the investor receives with probability one in the worst state net of monitoring costs. In other words, when bankruptcy occurs, on average the investor can recover at least $\underline{y}$ and expected enforcement costs.

Theorem 1. Simple debt contracts that are optimal (i.e., give the investor the highest payoff) among the class of deterministic contracts in the costly state verification

[7] Additive costs, $\bar{x} - c_E$, are chosen for simplicity. The results also hold for other cost structures (e.g., where costs are large or a percentage of total assets).

model and that satisfy technical conditions A.1, A.2 and the reservation utility constraint in Problem KV, are optimal in the more general costly enforcement model even when stochastic monitoring is possible.[8]

By showing that simple debt contracts are robust to stochastic monitoring, Krasa and Villamil resolve the well known Mookherjee and Png [10] critique that when stochastic monitoring is possible in the costly state verification model, simple debt contracts are no longer optimal. Theorem 1 establishes that simple debt is optimal when commitment is limited (i.e., agents can revise the promises made at $t = 0$ to $(v', \ell', e(v)')$). Consider now the relationship between deterministic contracts and debt.

- A deterministic contract means that the choice of v or e is deterministic (i.e., occurs with probability 1 or 0).
- Debt is a pair $(R(y), B)$ where y is the borrower's privately observed project outcome, $R(y)$ is a payment function, and B is a lower interval set of "bankruptcy states" where assets are seized. In a SDC, bankruptcy occurs with probability 1 or 0 as a deterministic function of y.[9]

A SDC $(R(y), B)$ is depicted below.

- $B = \{y < y*\}$: Lower interval bankruptcy set - bankruptcy occurs only for low project outcomes (i.e., below $y*$).
- $R(y) = y$: When bankruptcy occurs, the entire realization y is seized from the borrower.
- $R(y) = \bar{R}$: For all other sufficiently high realizations ($y \geq y*$), bankruptcy does not occur. The borrower makes a fixed payment $\bar{R}$ and retains $y - \bar{R}$.

In the Costly Enforcement model, contract $(v, \ell, v(y), e(v))$ is a SDC if there is a set of bankruptcy states B and a critical value y^* where

[8]The costly enforcement model extends the Townsend (1979) costly state verification model in two ways: (i) enforcement is a decision variable, and (ii) agents can renegotiate contracts. The precise definition of optimality was not explicit in Theorem 1 in Krasa and Villamil [8]. Sharma [11] pointed out that contracts which give the investor the highest payoff are necessary for the result to hold and that assumption (A2) had a typographical error in Krasa and Villamil [8].

[9]If state y were known, then bankruptcy occurs with probability 1 when the state is "bad" and with probability 0 when the state is "good."

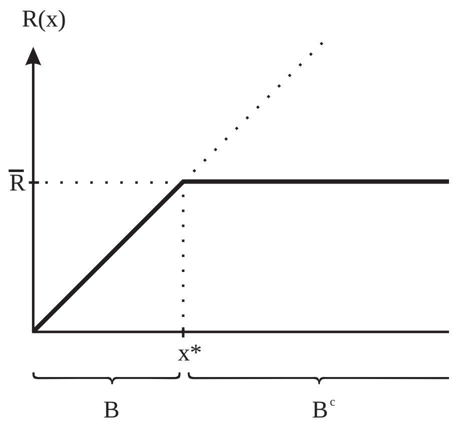

Figure 1: Simple Debt Contact

1. $B = \{y \in Y \mid y < y^*\}$, and the lender's enforcement action is $e = 1$ for $y \in B$ and $e = 0$ for $y \in B^c$ with probability 1.

2. The borrower pays $v = \bar{R}$ for $y \in B^c$ and $v = 0$ for $y \in B$. Moreover, $\ell(x, v) = \begin{cases} y, & \text{for } y \in B; \\ 0, & \text{for } y \in B^c. \end{cases}$

This definition of a SDC corresponds to the Figure, with $R(\cdot)$ and B modified to accommodate limited commitment to payment and enforcement decisions: (1) requires enforcement to occur on a lower interval and (2) requires all assets to be seized when bankruptcy occurs.

When contracts are restricted to be deterministic a priori, Townsend [12], Gale-Hellwig [5] and Williamson [13] showed that debt is optimal in a CSV model. When monitoring is assumed to be deterministic, simple debt is optimal because it minimizes monitoring costs. However, as you saw in Home Work 3 in Lecture Note 1, in the CSV model debt is dominated by a contract with random monitoring when stochastic contracts are allowed. Thus in the CSV model debt

- is not robust to stochastic monitoring,
- is not ex-post efficient, and
- stochastic and deterministic contracts cannot co-exist.

In the Costly Enforcement model, the environment and choice variables differ from the CSV model. As a consequence, conditions exist under which debt is optimal but not subject to the problems noted above. Payment schedule $R(y) = v(y) + e\ell(\cdot)$ allows for

- voluntary and enforceable payments, and
- limited commitment to initial promises.

Problem KV shows that the decisions to make a voluntary payment or enforce ℓ are chosen as part of the contract and hence are optimal each period by construction. These strategies are not restricted to be deterministic at the outset. Rather, Theorem 1 proves that if commitment is limited (a friction imbedded in the primitives of the model), debt is optimal even when stochastic contracts are allowed.

The intuition for the result is as follows. Debt is optimal in the Costly Enforcement model because it minimizes information revelation. Renegotiation, and hence constraint (1.4), is only relevant when there is new information *and* agents have the opportunity to use this information to alter the initial contract. Debt weakens the incentive to renegotiate (i.e., constraint (1.4)) by minimizing information revelation via fixed payments in good states.

Krasa and Villamil [8] also derive conditions under which stochastic contracts are optimal (cf., Theorem 2). In these lectures we focus on deterministic contracts because they resemble debt. Whether deterministic or stochastic contracts are "more reasonable" depends on the underlying economic environment. The point is to show that changing agents' ability to commit to the initial contract affects the optimal contract. When commitment is limited, and the other conditions of the model hold, deterministic contracts are optimal. When agents are able to commit, and the conditions of Krasa and Villamil [8] Theorem 2 hold, stochastic contracts are optimal. A contract is optimal given a primitive economic environment.

3 Enforcement and Firm Finance

How does the nature of the enforcement technology affect firm finance? Krasa, Sharma and Villamil [7] characterize the court by two parameters, c and η, and permit the entrepreneur to have some assets, $1-\alpha$ units of input. The goal of the analysis is to determine how these parameters affect firm finance, both qualitatively and quantitatively.[10] In this section *we focus on pure strategy equilibria* for simplicity.

[10] In this model there is no court detection error, $\bar{x}$. Rather, η is the percent of entrepreneur assets that the court cannot seize.

Consider an economy with a risk-neutral entrepreneur and a risk-neutral lender. The entrepreneur owns a production technology that requires 1 unit of input to produce a random output Y with realization $y \in Y = \{\underline{y}, ..., \bar{y}\}$. Ex-ante the agents have a common prior $\beta(.)$ over Y, where $\beta(\cdot)$ has a probability density function $f(y)$. The entrepreneur has only $0 \leq 1 - \alpha < 1$ units of input, and must therefore borrow α units from the lender.[11]

The timing of events is as follows:

t=0 Agents specify an *enforceable loan contract* $\ell(\cdot)$, which is a payment schedule with state y determined by a court at $t = 2$, and payment $\upsilon \geq 0$ made by the entrepreneur at $t = 1$. If agents cannot agree, no loan is made.

t=1 The entrepreneur, but not the lender, privately observes project realization y and selects a payment $\upsilon \geq 0$. Payment υ is not enforceable by the court (though enforceable payment $\ell(\cdot)$ depends on υ), but cannot be retracted once made. Because υ is not enforceable, we refer to it as a *voluntary payment.*

t=2 The lender chooses whether to request costly enforcement by the court. If no enforcement is requested, the lender's payoff is υ and the entrepreneur's payoff is $y - \upsilon$. If enforcement is requested, the lender pays cost c and payment $\ell(\cdot)$ is transferred to the lender.

- lender payoff is $\upsilon + \ell(\cdot) - c$
- entrepreneur payoff is $y - [\upsilon + \ell(\cdot)]$.

Enforcement Technology: We focus on two parameters to describe enforcement:

- c: legal fees, court cost, accounting standards, corruption
- η: % of total entrepreneur assets the court cannot seize (creditor versus debtor protection)[12]

Thus, η determines the legal payments the court will enforce; the maximum enforceable payment is $(1 - \eta)(y - \upsilon)$.

[11] Because we will prove that the lender uses debt contracts, α is the percent of debt finance.

[12] η is affected by factors such as the exemptions specified in the bankruptcy code, inflation, and the length of bankruptcy proceedings. The higher these factors are, the higher η, which means that creditor protection is weak (equivalently, debtor protection is strong).

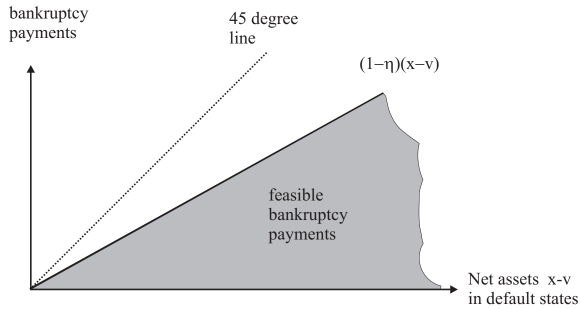

Figure 2: Feasible Bankruptcy Payments

The Figure illustrates the effect of the legal system on contract payments. Suppose that the entrepreneur repays nothing (i.e., $\upsilon = 0$) and the lender requests enforcement. The shaded, cone-shaped area is the set of all feasible bankruptcy payments. The court cannot seize η % of entrepreneur assets:

- The maximum possible payment to the lender is $(1 - \eta)y$.
- By an appropriate choice of ℓ, any payment in the cone can be obtained.

The figure illustrates that bankruptcy codes "protect" the debtor from paying more than $(1 - \eta)(y - \upsilon)$. Even if a contract specified a larger payment, the legally enforced payment must be in the cone.

Definition 2 *Payment schedule $\ell(\cdot)$ is legally enforceable if, for all y, υ with $y \geq \upsilon$, $0 \leq \ell(\cdot) \leq (1 - \eta)(y - \upsilon)$.*

3.1 The Investment Problem

The investment problem is a dynamic game of incomplete information because payoffs are vary endogenously as information changes during the game. We focus on pure strategy equilibria that are Pareto efficient in the set of all PBNE of the game. In the contract problem the planner chooses:

- $\boldsymbol{\upsilon}(y)$: an the entrepreneur's strategy for choosing voluntary payment υ.
- $\ell(\cdot)$: the legally enforceable payment function.

- $e(v)$: the lender's strategy for choosing whether to request the court to enforce $\ell(\cdot)$. If $e(v) = 1$ the lender requests enforcement of $\ell(\cdot)$ and if $e(v) = 0$ the lender does not request enforcement.

- $\beta(y|v)$: the lender's updated belief about the return at $t = 2$.

At first glance, it may seem unusual to specify beliefs as part of the contract problem. This natural extension of the well established Pareto approach allows for dynamic information revelation. In the contract literature it is standard to assume ex-ante (before information is revealed) that a "planner" coordinates agents on actions and a contract to attain an efficient allocation, subject to constraints. Krasa, Sharma and Villamil [7] also consider a planner who coordinates agents to achieve efficient outcomes, but the lender's off-equilibrium path beliefs $\beta(y|v)$ matter in the dynamic game because different beliefs give rise to different equilibrium payoffs. Thus, the planner must coordinate agents on payment function $\ell(\cdot)$, strategies $v(x)$ and $e(v)$ where payment v can reveal information, and beliefs that could arise if the entrepreneur were to deviate from the equilibrium strategy.[13]

Problem 1 *At $t = 0$, choose $\{v(y),\ \ell(\cdot),\ e(v),\ \beta(y|v)\ \}$ to maximize*

$$E_0[u_L(y)] = \int \Big[v(y) + e(v(y))\Big(\ell(y, v(y))\Big) - c\Big)\Big]\, d\beta(y) \tag{1}$$

[13] Many different off equilibrium path beliefs $\beta(y|v)$ support efficient outcomes. Each equilibrium consists of a strategy profile and a belief system, but agents' payoffs in all of these equilibria are the same. This approach differs from the refinements literature in game theory which imposes restrictions on beliefs (e.g., intuitive, divine, etc.). Instead, Krasa, Sharma and Villamil [7] admit any belief that supports an allocation on the Pareto frontier (where payoffs are maximized) of equilibria. Some refinement criteria may provide equilibria where the lender gets a lower payoff; this will not occur under this approach. In the problem below, constraint (5) pins down equilibrium beliefs via Bayes' rule and only off-equilibrium path beliefs must be chosen.

subject to

$$E_0[u_E(y)] = \int \Big[y - \mathscr{v}(y) - \mathscr{e}\big(\mathscr{v}(y)\big)\ell\big(y, \mathscr{v}(y)\big) \Big]\, d\beta(y) \geq \bar{u}_E \tag{2}$$

$$\mathscr{v}(y) \in \arg\max_{v \geq 0} \Big[y - v - \mathscr{e}\big(\mathscr{v}(y)\big)\ell\big(y, \mathscr{v}(y)\big) \Big] \tag{3}$$

$$\mathscr{e}(v) = 1 \text{ if and only if } \int \big[\ell(y, v) - c\big]\, d\beta(y|v) \geq 0 \tag{4}$$

$$\beta(y|v) \text{ is derived from } \beta(y) \text{ using Bayes' rule whenever possible} \tag{5}$$

$$\ell(\cdot) \text{ is enforceable} \tag{6}$$

This Problem maximizes the investor's payoff subject to the following constraints. (2) reflects voluntary participation by the entrepreneur, where $\bar{u}_E$ is a reservation utility.[14] (3) requires the entrepreneur's payment choice v at $t = 1$ to be optimal, i.e., maximize utility given realization y. (4) requires the lender's enforcement choice to be optimal, and (5) requires beliefs to be consistent. Thus, (3)–(5) require the strategies $\mathscr{v}$, $\mathscr{e}$ and beliefs $\beta(\cdot|v)$ to be a perfect Bayesian Nash equilibrium (PBNE). Finally, (6) requires payment $\ell(y, v)$ to be enforceable as specified in Definition 2.

3.2 The Equilibrium Contract: SDC

Krasa, Sharma and Villamil [7] show that a simple debt contract solves problem 1. The key characteristic of simple debt is that when enforcement occurs all possible assets are transferred, $(1 - \eta)(y - v)$, up to the amount owed, $\bar{v}$. Let y^* be the lowest non-bankruptcy state.

Definition 3 *$\{\ell(y, v), \mathscr{v}(y)\}$ is a simple debt contract if there exists $\bar{v}$ and $y^* \in [\underline{y}, \bar{y}]$ with $y^* \geq \bar{v}$ such that*

$$\ell(y, v) = \begin{cases} \min\{(1-\eta)y, \bar{v}\} & \text{if } y < y^*,\ v = 0; \\ 0 & \text{if } v \geq \bar{v}; \\ (1-\eta)(y - v) & \text{otherwise;} \end{cases} \qquad \mathscr{v}(y) = \begin{cases} \bar{v} & \text{if } y \geq y^*; \\ 0 & \text{if } y < y^*; \end{cases}$$

[14] (1) and (2) are equivalent to maximizing a weighted sum of the two agents' utilities. Varying the Pareto weights (or the reservation utility) gives the entire Pareto frontier.

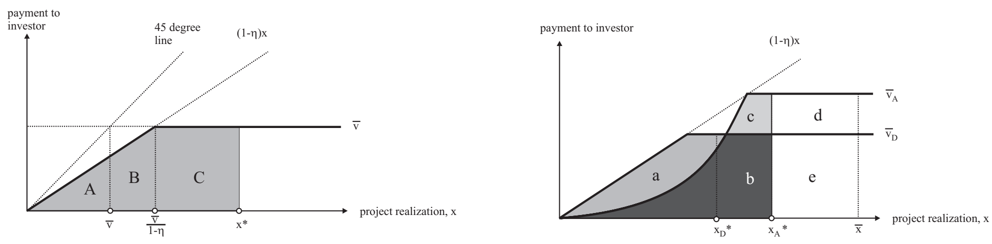

Figure 3: Simple Debt Contracts with Costly Enforcement and Exemptions

The main difference between the CE model and the standard CSV model is generated by sequential rationality – the investor must be willing to enforce when the entrepreneur defaults. In the CSV model the sole concern is to minimize expected bankruptcy costs; there is no need to provide an incentive to enforce and default occurs if and only if the entrepreneur is unable to pay (Gale and Hellwig [5], Townsend [12] or Williamson [14]). In contrast, the CE model accommodates both inability to pay and willful default. An example illustrates the intuition.

Assume that a debtor

- Owes $\bar{v} = \$100,000$,
- Has assets: home equity of $\$50,000$, private property of $\$80,000$, and retirement savings of $\$100,000$.

The total value of assets, $y = \$230,000$, exceeds $\bar{v}$.

Example. If the debtor files for bankruptcy in Texas, under state law all equity in a homestead and pension/retirement accounts are exempt, as is personal property up to $60,000. Chapter 7 specifies that exempt assets cannot be used to satisfy creditor claims. As a consequence, the court can only seize $(1 - \eta)y = \$20,000$. This amount is transferred to the creditors (net of c) and the case is discharged by the court. The debtor is "protected" from paying the remaining $\$80,000$.

Point: Given a particular bankruptcy code, it may be optimal for a debtor to default. Default is "willful" whenever total assets y exceed the amount owed $\bar{v}$, but the bankruptcy code protects the debtor from judgments against the exempt portion of assets.

The first panel of Figure 3 illustrates the default regions in a simple debt contract when $\underline{y} = 0$. Inability to pay occurs in area A where debtor assets are less than the amount owed, $y < \bar{v}$. Willful default occurs when the debtor defaults but total assets, y, are sufficient to pay $\bar{v}$. This occurs when $y > \bar{v}$ for two distinct reasons. First, because η percent of the debtor's assets are not seized, the debtor will not repay if $(1 - \eta)y \leq \bar{v}$, i.e., if $y \leq \frac{\bar{v}}{1-\eta}$. This is area B. Second, constraint (4) may generate an additional willful default region C. Constraint (4) gives the lender the incentive to request enforcement if the entrepreneur does not pay $\bar{v}$ and requires the lender's enforcement decision to be sequentially rational, i.e., the lender's expected enforcement payoff must cover cost c. Region C will arise if regions A and B are not sufficient to cover cost c.[15] In the efficient equilibria characterized by problem 1, the entrepreneur will announce default in just enough states to induce the lender to enforce when the entrepreneur defaults, i.e., (4) must hold. This adds region C with bankruptcy states $\frac{\bar{v}}{1-\eta} \leq y < y^*$ when (4) binds. Finally, constraint (3) ensures that the entrepreneur will default when the lender expects default to occur, i.e., $\ell(y, v) \leq \bar{v}$. In a simple debt contract it is optimal to make the bankruptcy payment as large as possible. Therefore, $\ell(y, v) = \bar{v}$ in region C.

The second panel of Figure 3 illustrates the intuition the result that SDCs are optimal. Consider a simple debt contract with face value $\bar{v}_D$ and an arbitrary debt contract with face value $\bar{v}_A$.[16] The payment the lender expects under contract $\bar{v}_A$ is area $b + c + d + e$. Find a simple debt contract with face value $\bar{v}_D$ such that the lender's expected payment is the same as under the original contract, i.e., $a + b + e = b + c + d + e$. This implies that $a + b > b + c$, where $b + c$ is the bankruptcy area under the alternative contract and $a+b$ is the bankruptcy area under the simple debt contract. If bankruptcy occurs for all states $y < y_A^*$ in both contracts, then the lender's expected bankruptcy payment is strictly higher under simple debt contract $\bar{v}_D$. This implies that constraint (4) is slack. The size of the bankruptcy set for the simple debt contract can then be reduced to y_D^*, thereby decreasing expected enforcement costs, which increases the lender's total expected payoff.

Krasa, Sharma and Villamil [7] show that we can use the fact that simple debt is the optimal contract to simplify problem 1 as follows.

[15] If the lender expects the entrepreneur to default only if $y < \frac{\bar{v}}{1-\eta}$, the lender will never enforce. This implies that the entrepreneur has the incentive to default in additional states.

[16] In the proof, contract v_A need not be debt.

Problem 2 *At $t = 0$, choose $\bar{v}$ and y^* to maximize*

$$E_0[u_L(y)] = \int_{\underline{y}}^{\frac{\bar{v}}{1-\eta}} (1-\eta) y \, d\beta(y) + \int_{\frac{\bar{v}}{1-\eta}}^{\bar{y}} \bar{v} \, d\beta(y) - \int_{\underline{y}}^{y^*} c \, d\beta(y) \tag{7}$$

subject to

$$E_0[u_E(y)] = \int_{\underline{y}}^{\frac{\bar{v}}{1-\eta}} \eta y \, d\beta(y) + \int_{\frac{\bar{v}}{1-\eta}}^{\bar{y}} (y - \bar{v}) \, d\beta(y) \geq \bar{u}_E \tag{8}$$

$$\frac{\bar{v}}{1-\eta} \leq y^* \tag{9}$$

$$\int_{\underline{y}}^{\frac{\bar{v}}{1-\eta}} (1-\eta) y \, d\beta(y | y < y^*) + \int_{\frac{\bar{v}}{1-\eta}}^{y^*} \bar{v} \, d\beta(y | y < y^*) - c \geq 0 \tag{10}$$

$$(1-\eta)(\bar{y} - \bar{v}) - c \geq 0 \tag{11}$$

Objective (7) and constraint (8) correspond to (1) and (2) of problem 1. Constraint (9) follows from (3) and specifies that default must occur at least in all states y with $y < \frac{\bar{v}}{1-\eta}$, which implies $\frac{\bar{v}}{1-\eta} \leq y^*$ (see figure 3). Constraint (4) implies (10) and (11), where (10) considers the case where payment occurs on the equilibrium path and (11) considers off-equilibrium path payments v. Because beliefs off the equilibrium path are optimistic as explained above, (4) implies $(1-\eta)(\bar{y} - v) - c \geq 0$ for all $v < \bar{v}$, which in turn implies (11). Finally, (5) and (6) of problem 1 are satisfied by construction. Existence of a solution follows from standard compactness and continuity arguments.

The dynamic enforcement game in the CE model is fundamentally different than the CSV model with full commitment because, as explained above, sequential rationality may require area C. Further, Figure 3 shows that η generates a "change in slope," and hence payoffs, in regions A and B. The change in payoffs can lead to important quantitative differences relative to the CSV model, but this effect does not require a dynamic game.

The CE model specifies the liquidation process and shows how parameters η and c affect the incentive to pursue bankruptcy. Agents may choose to enter bankruptcy even if they could pay, which is consistent with empirical observation.[17]

[17]Testimony before the House Judiciary Committee in 2002 indicated that in the U.S. "about 25% of Chapter 7 debtors could have repaid at least 30% of their non-housing debts over a 5-year

The CE approach differs from the literature on strategic default in incomplete contract models (cf., Anderson and Sundaresan [1] or Mella-Barral and Perraudin [9]) which are also dynamic games with renegotiation. The CE model is a stylized description of bankruptcy liquidation (US, Chapter 7), whereas in these models the legal authority solely assigns ownership rights. Bankruptcy is interpreted as a situation where "control" is transferred from the firm to creditors, and strategic default corresponds to debt forgiveness. The debtor is not "forced into costly bankruptcy" because it is Pareto improving for both parties to renegotiate to avoid costly liquidation.

Krasa, Sharma and Villamil [7] construct a model of enforcement where the legal system is described by parameters, η and c, and the lender has an incentive to request enforcement (which is ensured by constraints (10) and (11)). Theorems 1 and 2 provide complete characterizations of the effect of enforcement parameters c and η, respectively, on the bankruptcy probability and the interest rate. The face value and interest rate are related by $\bar{v} = \alpha(1 + r)$ and the bankruptcy probability is $\beta[\underline{y}, y^*]$.

3.3 Efficiency of the Court: *c*

Theorem 1 analyzes the effect of c on finance. The size of c measures the efficiency of bankruptcy procedures. Assume that $\beta(y)$ has a density function $f(y)$ that is differentiable.

Theorem 1

1. *Assume that c is increased. Then the lender's expected payoff is decreased. The decrease is strict if the bankruptcy probability is strictly positive.*

2. *When c changes, the effect on the interest rate and the bankruptcy probability is characterized by four distinct parameter regions.*

   Region 1 *If (8) binds, but (10) and (11) do not bind, which occurs for small c, the interest rate and the bankruptcy probability do not depend on c.*

   Region 2 *If (8), (10) and (11) do not bind, which may occur for intermediate values of c, the bankruptcy probability and the interest rate are decreasing in c.*

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repayment plan, after accounting for monthly expenses and housing payments" and that "about 5% of Chapter 7 filers appeared capable of repaying all of their non-housing debt over a 5-year plan." Under Chapter 7, this debt is extinguished and need never be repaid.

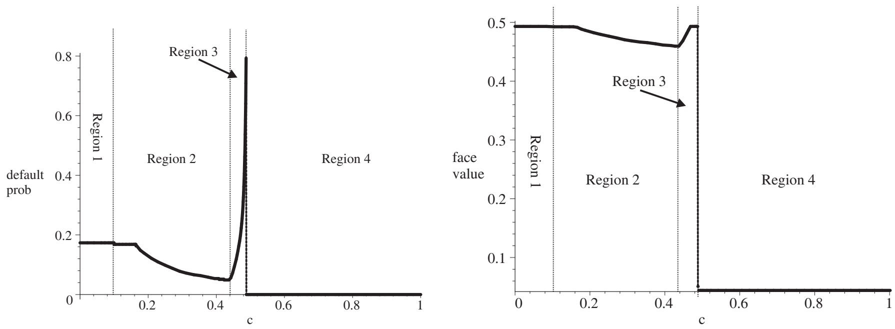

Figure 4: The Four Regions of Theorem 1

Region 3 *If (10) binds but (11) does not bind, which occurs for larger values of c, the interest rate and bankruptcy probability increase. If (8) holds with equality, the interest rate is constant.*

Region 4 *If c is sufficiently large, the bankruptcy probability is zero. The interest rate is constant, unless (11) binds, in which case it decreases.*

Figure 4 illustrates Theorem 1 for baseline parameter values.[18] In region 1, the entrepreneur's participation constraint binds. Therefore, the face value does not change with c. This, in turn, means that the bankruptcy probability remains constant. In region 2, c is sufficiently high that it becomes optimal to reduce the face value $\bar{v}$. Reducing $\bar{v}$ reduces the bankruptcy probability and saves expected bankruptcy costs. For the lender, this saving compensates for the lower face value. In region 3, (10) binds. This means that y^* must be increased to give the lender an incentive to enforce. Once c is sufficiently large it becomes optimal not to provide finance, or to invest solely in projects that are fully collateralized, i.e., where $\underline{y} >> 0$. The inability of entrepreneurs to obtain finance is a significant problem in many emerging markets. Our result indicates that high enforcement costs can easily be a source of credit market failure. In practice, cost c includes payments to accountants, lawyers, and the court to establish the size of the entrepreneur's assets, y, and bribes to expedite the case or influence the outcome. The government can play an important role in determining the size of c by requiring a high level of disclosure and routine accounting practices, and by policies to deter corruption.

[18]The baseline parameter values are $\alpha = 0.5$, $\bar{u}_E = 0.1$, $\eta = 0$ and $f(y)$ is a lognormal distribution. We will evaluate parameter sensitivity in the next section.

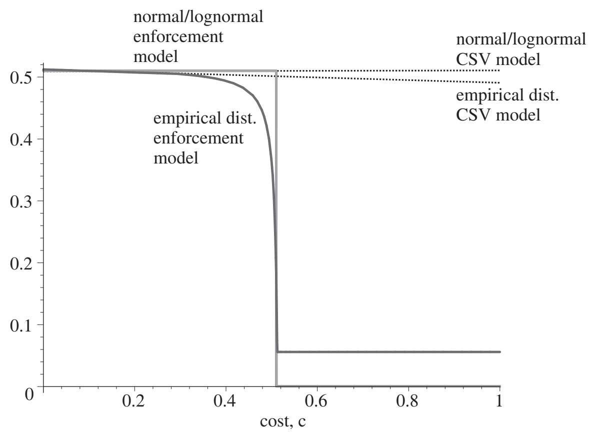

Figure 5: The Effect of c on the Lender's Expected Payoff

Figure 5 shows how the lender's expected payoff varies with enforcement cost c. The most striking result is that there are two regions over which changes in c have almost no effect (these correspond to regions 1 and 4 in Theorem 1).[19] Intermediate region 3 is small and highly sensitive to small changes in c. The transition between region 1, where finance occurs, and region 4 where finance is severely compromised is especially rapid for the lognormal distribution. The intuition for this is as follows. Constraint (10) does not bind for some sufficiently low $\underline{c}$ which characterizes region 1. Next, note that (10) cannot hold if c is strictly greater than the face value $\bar{v}$. Thus there is a critical $\bar{c}$ that determines region 4. The transition between regions 1 and 4 consists of all c between $\underline{c}$ and $\bar{c}$.

Theorem 1 and Figure 5 are interesting because they indicate that countries with poor institutions may experience rapid and severe "financial crises" due to a small change in fundamentals, c, such as a bribery or accounting scandal. Theorem 1 predicts that this phenomenon would not be observed in low cost countries (e.g., the U.S.), but would be observed in intermediate cost countries. Countries with high costs (e.g., sub-Saharan Africa) would have low expected returns, and therefore would receive little private investment unless c was lowered substantially. Finally, Figure 5 illustrates that the predictions of our dynamic enforcement game can differ markedly from those of the CSV model.

[19]In this example region 2 from Theorem 1 does not occur.

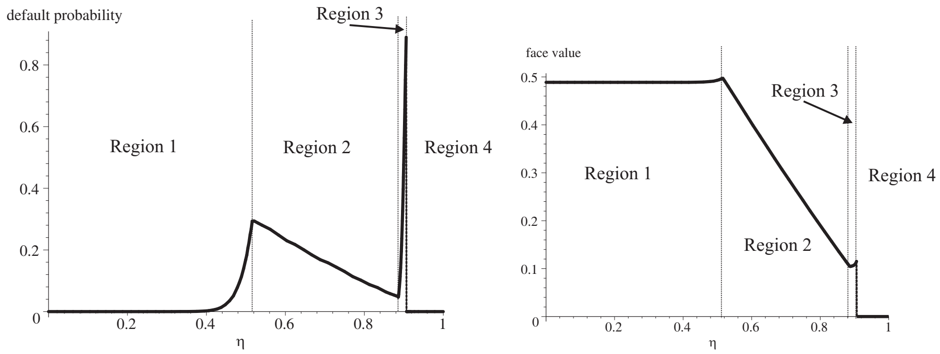

Figure 6: The Four Regions of Theorem 2

3.4 The Effect of Exemptions and Inflation: η

Parameter η determines the percent of total assets that the court cannot seize due to exemptions in the legal code or because inflation lowers the real value of creditor claims. Theorem 2 investigates the impact of η on the optimal contract. The result follows because increasing η decreases the cone of feasible payments (see figure 2).

Theorem 2

1. *Assume that η is increased. Then the lender's expected payoff decreases. The decrease is strict if $c > 0$ and if bankruptcy occurs with positive probability.*

2. *When η changes, the effect on the interest rate and the bankruptcy probability is characterized by four distinct parameter regions.*

   Region 1 *If (8) binds but (10) and (11) do not, which occurs if η and c are not too large, the interest rate and bankruptcy probability are increasing in η.*

   Region 2 *If (8), (10) and (11) do not bind, which occurs for intermediate values of η, the interest rate and bankruptcy probability are decreasing in η.*

   Region 3 *If (10) binds, which occurs for larger values of η, the bankruptcy probability is increasing in η. The interest rate is increasing in η if (8) also binds.*

   Region 4 *If η is sufficiently close to 1, the bankruptcy probability is 0. The interest rate is constant unless (11) binds, in which case it decreases.*

Figure 6 illustrates Theorem 2. In the graphs we show how the bankruptcy probability and the contract's face value vary with η for the baseline parameters. The face value is $\bar{v} = \alpha(1 + r)$. The intuition for each region is as follows.

In region 1 as η increases, the entrepreneur retains more assets in bankruptcy. In order to make up for this, the lender raises the face value. In the graph the increase in the face value is small until η is close to region 2. In contrast, the increase in the bankruptcy probability is more rapid because increasing the face value has direct and indirect effects on bankruptcy – as η increases the entrepreneur enjoys more bankruptcy protection and this increases the entrepreneur's incentive to default.

In region 2 an increase in η, ceteris paribus, would lead to a further increase in the bankruptcy probability. However, at the end of region 1 it is inefficient to increase the bankruptcy probability further because expected bankruptcy costs are large. In order to keep the bankruptcy probability at least constant, the face value must be decreased.[20] However, as η gets larger it becomes optimal to actually decrease the bankruptcy probability. Recall figure 3. In region 2, $y^* = \frac{\bar{v}}{1-\eta}$. Moreover, at the optimum the marginal loss to the lender of lowering the face value by $\Delta\bar{v}$ must equal the marginal gain of a decreased bankruptcy probability. If $\bar{v}$ is decreased by $\Delta\bar{v}$, then y^* decreases by $\Delta\bar{v}/(1 - \eta)$, which is the lender's gain from less bankruptcy. This benefit increases as η increases. Therefore, a larger η results in a lower y^* and hence a lower bankruptcy probability. This decrease of y^* accelerates the drop in the face value because to keep the bankruptcy probability constant, we must lower $\bar{v}$. Hence, to lower the bankruptcy probability, $\bar{v}$ must decline at an even faster rate.

Region 3 occurs when η is relatively large and (10) binds. In figure 3 this means that y^* is increased. The bankruptcy probability quickly increases to a level where it is no longer optimal to provide finance, which leads to region 4.

How large might η be? Suppose $\eta = 1/2$. Recall that under bankruptcy law in Texas all equity in a homestead and pension/retirement accounts are exempt. Suppose, for example, that a court assesses that a debtor has \$1 million in assets. If homestead equity is \$200,000 and retirement savings are \$300,000, then the value for η is 50%.[21] The second example applies to countries with inflation and bankruptcy delays where contracts cannot be indexed for inflation (as in Mexico before 1996). If a bankruptcy case is expected to take 6 years (as was the case in Mexico), then a steady inflation rate of 11% compounded over 6 years lowers the

[20] Recall the argument for region 1 that increasing η, keeping the face value constant, increases the bankruptcy probability.

[21] If assets can be concealed, then η can be even higher.

value of creditor claims by about 50%. The actual average inflation rate of 16.6% reported by the Banco de México for the last 10 years clearly indicates that the drop-off is a legitimate concern in many economies.[22]

[22] Boyd, Levine and Smith [2] find empirical evidence of an inflation threshold of 15% (for economies with inflation rates exceeding 15%, there is a discrete drop in financial sector performance). This corresponds to the non-linearity in our theoretical model at a critical η. The value of η implied by an inflation rate of π over n years is given by $1 - \eta = (1 - \pi)^n$.

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