A Topological Problem in Magnetohydrodynamics

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Abstract

In magnetohydrodynamics, a magnetic field **B** evolves in time with no change in the topology $\mathcal{T}(\mathbf{B})$ of its lines of force defined by

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}.$$

In this physical sense, a field in static equilibrium governed by the force-free equations:

$$\nabla \times \mathbf{B} = \alpha\ \mathbf{B}$$
$$\nabla \cdot \mathbf{B} = 0,$$

is subject to a prescribed $\mathcal{T}(\mathbf{B})$ the field happens to possess from its evolutionary past. I will describe the challenge of expressing $\mathcal{T}(\mathbf{B})$ as integral equations, the mixed (hyperbolic & elliptic) nature of the force-free pdes, and, the general need for this nonlinear, nonlocal problem to admit weak solutions in which **B** is tangentially discontinuous at suitably placed surfaces. It is instructive to contrast this kind of weak solutions with those containing shocks associated with intersecting characteristics in compressive hydrodynamics as a purely hyperbolic pde system. The construction of such equilibrium states by numerical simulation, via 3D time-dependent topology-preserving relaxation, will be briefly discussed, pointing out an interesting break through. The formation of magnetic discontinuities as proposed by E. N. Parker is a promising explanation of the million-degree temperature of the outer atmosphere of the Sun.

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