ON STOCHASTIC INTEGRAL REPRESENTATION OF SOME PARTIAL MAX-FUNCTIONALS OF THE BROWNIAN MOTION

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It is well known that the Itô–Clark theorem gives the stochastic integral representation for every square integrable functional $S = S(\omega)$ of a given Brownian motion $B = (B_t(\omega))_{t<\infty}$ in a form

$$S = C + \int_0^\infty H_s \, dB_s.$$

However, the explicit form of the integrant $H = (H_s)_{s\geq 0}$, as a rule, is difficult to find.

In this talk we present our results obtained together with M. Yor which concern stochastic integral representation for the following functionals:

$$\max_{u\leq T} B_u, \quad \max_{u\leq T^b_{-a}} B_u, \quad \max_{u\leq \theta_T} B_u, \quad \max_{u\leq g_T} B_u,$$

where

$$T^b_{-a} = \inf\{t \geq 0 : B_t \notin (-a, b)\}, \quad a > 0, \;\; b > 0,$$
$$\theta_T = \sup\Big\{t \leq T : B_t \min_{s\leq t} B_s\Big\},$$
$$g_T = \sup\{t \leq T : B_t = 0\}.$$

(Notice that times θ_T and g_T are not Markovian.)