

Geometry of multiplicity-free representations of $GL(n)$, visible actions on flag varieties, and triunity

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Abstract

We analyze the criterion of the multiplicity-free theorem of representations [5, 6] and explain its generalization. The criterion is given by means of geometric conditions on an equivariant holomorphic vector bundle, namely, the “visibility” of the action on a base space and the multiplicity-free property on a fiber.

Then, several finite dimensional examples are presented to illustrate the general multiplicity-free theorem, in particular, explaining that three multiplicity-free results stem readily from a single geometry in our framework. Furthermore, we prove that an elementary geometric result on Grassmann varieties and a small number of multiplicity-free results give rise to all the cases of multiplicity-free tensor product representations of $GL(n, \mathbb{C})$, for which Stembridge [12] has recently classified by completely different and combinatorial methods.

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