Variational Image Deblurring - A Window into Mathematical Image Processing

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To recover a sharp image from its blurry observation is the problem known as *image deblurring*. It frequently arises in imaging sciences and technologies, including optical, medical, and astronomical applications, and is a crucial step towards successful detections of important patterns such as abnormal tissues and surface details of a distant planet.

Mathematically, image deblurring is intimately connected to backward diffusion processes (e.g., inverting the heat equation), which are notoriously unstable. As inverse problem solvers, deblurring models therefore crucially depend upon proper regularizers or conditioners that help secure stability, often at the necessary cost of losing certain high-frequency details in original clear images. Such regularization techniques can ensure the existence or uniqueness of deblurred images.

In this paper, we present a comprehensive theory of image deblurring by half surveying the existent literature and half developing a systematic and general framework. Our novel efforts include presenting the physics foundations of common blurs, classifying deblurring problems, and developing both rigorous mathematical analysis on existence or uniqueness and the associated computational methods. In combination, the current work provides a window into the vast contemporary literature of mathematical image analysis and processing - its main modelling ideas and techniques, mathematical analysis involved, as well as major computational approaches.