RECIPROCITY ALGEBRAS AND BRANCHING FOR CLASSICAL SYMMETRIC PAIRS

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ABSTRACT. We study branching laws for a classical group G and a symmetric subgroup H. Our approach is through the *branching algebra*, the algebra of covariants for H in the regular functions on the natural torus bundle over the flag manifold for G. We give concrete descriptions of (natural subalgebras of) the branching algebra using classical invariant theory. In this context, it turns out that the ten classes of classical symmetric pairs (G, H) are associated in pairs, (G, H) and (H', G'), and that the (partial) branching algebra for (G, H) also describes a branching law from H' to G'. (However, the second branching law may involve certain infinite-dimensional highest weight modules for H'.) To highlight the fact that these algebras describe two branching laws simultaneously, we call them *reciprocity algebras*. Our description of the reciprocity algebras reveals that they all are related to the tensor product algebra for GL_n. This relation is especially strong in the *stable range*. We give quite explicit descriptions of reciprocity algebras in the stable range in terms of the tensor product algebra for GL_n. This is the structure lying behind formulas for branching multiplicities in terms of Littlewood-Richardson coefficients.

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