

On a generalized dimension of self-affine fractals

by

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Abstract

For a $d \times d$ expanding matrix A , we define a pseudo-norm $w(x)$ in terms of A and use this pseudo-norm (instead of the Euclidean norm) to define the Hausdorff measure and the Hausdorff dimension $\dim_H^w E$ for subsets E in \mathbb{R}^d . We show that this new approach gives convenient estimations to the classical Hausdorff dimension $\dim_H E$, and in the case that the eigenvalues of A have the same modulus, then $\dim_H^w E$ and $\dim_H E$ coincide. This setup is particularly useful to study self-affine sets T generated by $\phi_j(x) = A^{-1}(x + d_j)$, $d_j \in \mathbb{R}^d$, $j = 1, \dots, N$. We use it to investigate the fractality of T for the case that $\{\phi_j\}_{j=1}^N$ satisfies the open set condition as well as the cases without the open set condition. We extend some well known results in the self-similar sets to the self-affine sets.

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