WAVELETS FROM THE LOOP SCHEME

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Abstract. A new wavelet-based geometric mesh compression algorithm was developed recently in the area of computer graphics by Khodakovsky, Schröder, and Sweldens in their interesting paper [22]. The new wavelets used in [22] were designed from the Loop scheme using ideas in [25, 26], where orthogonal wavelets with exponential decay and pre-wavelets with compact support were constructed. The wavelets have the same smoothness order as that of the basis function of the Loop scheme around the regular vertices which has a continuous second derivative, and have smaller supports than those wavelets obtained by constructions in [25, 26] or any other compactly supported biorthogonal wavelets derived from the Loop scheme (e.g. [10, 11]). Hence, the wavelets used in [22] have a good time frequency localization. This leads to a very efficient geometric mesh compression algorithm as proposed in [22]. As a result, the algorithm in [22] outperforms several available geometric mesh compression schemes used in the area of computer graphics. However, it remains open whether the shifts and dilations of the wavelets form a Riesz basis of $L_2(\mathbb{R}^2)$. Riesz property plays an important role in any wavelet-based compression algorithm and is critical for the stability of any wavelet-based numerical algorithms. We confirm here that the shifts and dilations of the wavelets used in [22], as expected, do indeed form a Riesz basis of $L_2(\mathbb{R}^2)$ by applying the more general theory established in this paper.

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