Determining Modes, Nodes and Volume Elements for Stationary Solutions to the Navier-Stokes Equations Past a Three-Dimensional Body

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Abstract

In this paper we show that every solution to the three-dimensional exterior Navier-Stokes boundary-value problem, corresponding to a given non-zero, constant velocity at infinity (flow past a body) and belonging to a very general functional class, $\mathcal{S}$, can be determined by a discrete number of parameters. Our results extend analogous classical results by Foiaş and Temam [6], [7], and by Jones and Titi [13] for the interior problem. This extension is by no means trivial, in that all fundamental tools used in the case of the interior problem –such as compactness of the Sobolev embeddings, Poincaré's inequality, and the special basis constituted by eigenfunctions of the Stokes operator– are no longer available for the exterior problem. An important consequence of our results is that any solution in $\mathcal{S}$ is uniquely determined by the knowledge of the associate velocity field only "near" the boundary. Just how "near" it has to be depends only on the Reynolds number and on the body.