

# General dynamical sharp-interface conditions for phase transformations in viscous heat-conducting fluids

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We develop a complete set of equations governing the evolution of a sharp interface separating two fluid phases undergoing transformation. In addition to the conventional balances for mass, linear momentum, and energy these equations include also a counterpart of the Gibbs–Thomson equation familiar from theories for crystal growth. This additional equation arises from a consideration of configurational forces within a thermodynamical framework. While the notion of configurational forces is well-developed and understood for the description of materials, like crystalline solids, that possess natural reference configurations, very little has been done regarding their role in materials, such as viscous fluids, that do not possess preferred reference states. We therefore provide a comprehensive discussion of configurational forces, the balance of configurational momentum, and configurational thermodynamics that does not require a choice of reference configuration. The general evolution equations arising from our theory account for the thermodynamic structure of the bulk phases and the interface and for viscous and thermal dissipation in the bulk phases and for viscous dissipation on the interface. Due to the complexity of these equations, we provide a reduced system of equations based on simplified constitutive assumptions and approximations common in the literature on phase transformations.

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