The Rate of Convergence of the Augmented Lagrangian Method for Nonlinear Semidefinite Programming*

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Abstract

We analyze the local convergence rate of the augmented Lagrangian method in nonlinear semidefinite optimization. The presence of the positive semidefinite cone constraint requires extensive tools such as the singular value decomposition of matrices, an implicit function theorem for semismooth functions, and variational analysis on the projection operator in the symmetric matrix space. Without requiring strict complementarity, we prove that, under the constraint nondegeneracy condition and the strong second order sufficient condition, the rate of convergence is linear and the ratio constant is proportional to $1/c$, where c is the penalty parameter that exceeds a threshold $\bar{c} > 0$.

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