

A Duality View of Spectral Methods for Dimensionality Reduction*

Lin Xiao[†] Jun Sun[‡] Stephen Boyd[§]

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Abstract

We present a unified duality view of several recently emerged spectral methods for nonlinear dimensionality reduction, including Isomap, locally linear embedding, Laplacian eigenmaps, and maximum variance unfolding. We discuss the duality theory for the maximum variance unfolding problem, and show that other methods are directly related to either its primal formulation or its dual formulation, or can be interpreted from the optimality conditions. This duality framework reveals close connections between these seemingly quite different algorithms. In particular, it resolves the myth about these methods in using either the top eigenvectors of a dense matrix, or the bottom eigenvectors of a sparse matrix — these two eigenspaces are exactly aligned at primal-dual optimality.

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[†]Center for the Mathematics of Information, California Institute of Technology, Pasadena, CA 91125, USA. Email: lxiao@caltech.edu.

[‡]Department of Electrical Engineering, Stanford University, Stanford, CA 94305, USA. Email: sunjun@stanford.edu.

[§]Department of Electrical Engineering, Stanford University, Stanford, CA 94305, USA. Email: boyd@stanford.edu.