

# ON THE ORBITS OF COMPUTABLE ENUMERABLE SETS

PETER A. CHOLAK, RODNEY DOWNEY, AND LEO A. HARRINGTON

ABSTRACT. The goal of this paper is to show there is a single orbit of the c.e. sets with inclusion,  $\mathcal{E}$ , such that the question of membership in this orbit is  $\Sigma_1^1$ -complete. This result and proof have a number of nice corollaries: the Scott rank of  $\mathcal{E}$  is  $\omega_1^{\text{CK}} + 1$ ; not all orbits are elementarily definable; there is no arithmetic description of all orbits of  $\mathcal{E}$ ; for all finite  $\alpha \geq 9$ , there is a properly  $\Delta_\alpha^0$  orbit (from the proof).

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