Approximation of the solution and its derivative for the singularly perturbed Black-Scholes equation with nonsmooth initial data*

Shuiying Li †, Grigory I. Shishkin and Lidia P. Shishkina ‡

A problem for the Black-Scholes equation that arises in financial mathematics, by a transformation of variables, is leaded to the Cauchy problem for a singularly perturbed parabolic equation with variables x, t and a perturbation parameter ε, $\varepsilon \in (0, 1]$. This problem has several singularities such as: the unbounded domain; the piecewise smooth initial function (its first order derivative in x has a discontinuity of the first kind at the point $x = 0$); an interior (moving in time) layer generated by the piecewise smooth initial function for small values of the parameter ε; etc.

In this paper, a grid approximation of the solution and its first order derivative is studied in a finite domain including the interior layer. On a uniform mesh, using the method of additive splitting of a singularity of the interior layer type, a special difference scheme is constructed that allows us to approximate ε-uniformly both the solution of the boundary value problem and its first order derivative in x with convergence orders close to 1 and 0.5, respectively. The efficiency of the constructed scheme is illustrated by numerical experiments.

*This research was supported in part by the NUS Academic Recearch Fund (grant No R-151-000-025-112), the Russian Foundation for Basic Research under grant No. 04-01-00578, 04–01–89007–NWO_a, by the Dutch Research Organisation NWO under grant No. 047.016.008 and by the Boole Centre for Research in Informatics, National University of Ireland, Cork.

†Department of Computational Science, National University of Singapore, Singapore 117543

‡Institute of Mathematics and Mechanics, Ural Division of RAS, Yekaterinburg, Russia