# MAJORITY BOOTSTRAP PERCOLATION ON THE HYPERCUBE

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ABSTRACT. In majority bootstrap percolation on a graph $G$, an infection spreads according to the following deterministic rule: if at least half of the neighbours of a vertex $v$ are already infected, then $v$ is also infected, and infected vertices remain infected forever. Percolation occurs if eventually every vertex is infected.

The elements of the set of initially infected vertices, $A \subset V(G)$, are normally chosen independently at random, each with probability $p$, say. This process has been extensively studied on the sequence of torus graphs $[n]^d$, for $n = 1, 2, \ldots$, where $d = d(n)$ is either fixed or a very slowly growing function of $n$. For example, Cerf and Manzo [14] showed that the critical probability is $o(1)$ if $d(n) \leqslant \log_* n$, i.e., if $p = p(n)$ is bounded away from zero then the probability of percolation on $[n]^d$ tends to one as $n \to \infty$.

In this paper we study the case when the growth of $d$ to $\infty$ is not excessively slow; in particular, we show that the critical probability is $1/2 + o(1)$ if $d \geqslant (\log \log n)^2 \log \log \log n$, and give much stronger bounds in the case that $G$ is the hypercube, $[2]^d$.