The Linus sequence

Paul Balister[*†] Steve Kalikow[*] Amites Sarkar[*†]

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Abstract

Define the Linus sequence L_n for $n \geq 1$ as a 0-1 sequence with $L_1 = 0$, and L_n chosen so as to minimize the length of the longest repeated block $L_{n-2r+1} \dots L_{n-r} = L_{n-r+1} \dots L_n$. Define the Sally sequence S_n as the length r of the longest repeated block that was avoided by the choice of L_n. We prove several results about these sequences, such as exponential decay of the frequency of highly periodic subwords of the Linus sequence, zero entropy of any stationary process obtained as a limit of word frequencies in the Linus sequence, and infinite average value of the Sally sequence. In addition we make a number of conjectures about both sequences.

[*]University of Memphis, Department of Mathematics, Dunn Hall, 3725 Norriswood, Memphis, TN 38152, USA

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