Negative magnetic susceptibility and nonequivalent ensembles for the mean-field ϕ^4 spin model

Alessandro Campa*
Complex Systems and Theoretical Physics Unit, Health and Technology Department, Istituto Superiore di Sanità and INFN Roma, Gruppo Collegato Sanità, Viale Regina Elena 299, 00161 Roma, Italy

Stefano Ruffo†
Dipartimento di Energetica "Sergio Stecco" and CSDC, Università di Firenze and INFN, Via S. Marta 3, 50139 Firenze, Italy

Hugo Touchette‡
School of Mathematical Sciences, Queen Mary, University of London, London E1 4NS, UK
(Dated: March 31, 2007)

We calculate the thermodynamic entropy of the mean-field ϕ^4 spin model in the microcanonical ensemble as a function of the energy and magnetization of the model. The entropy and its derivative are obtained from the theory of large deviations, as well as from Rugh's microcanonical formalism, which is implemented by computing averages of suitable observables in microcanonical molecular dynamics simulations. Our main finding is that the entropy is a concave function of the energy for all values of the magnetization, but is nonconcave as a function of the magnetization for some values of the energy. This last property implies that the magnetic susceptibility of the model can be negative when calculated microcanonically for fixed values of the energy and magnetization. This provides a magnetization analog of negative heat capacities, which are well-known to be associated in general with the nonequivalence of the microcanonical and canonical ensembles. Here, the two ensembles that are nonequivalent are the microcanonical ensemble in which the energy and magnetization are held fixed and the canonical ensemble in which the energy and magnetization are fixed only on average by fixing the temperature and magnetic field.

PACS numbers: 05.20.-y, 05.20.Gg, 05.70.Fh