

Cliques and the Spectral Radius

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Abstract

We prove a number of relations between the number of cliques of a graph G and the largest eigenvalue $\mu(G)$ of its adjacency matrix. In particular, writing $k_s(G)$ for the number of s -cliques of G , we show that, for all $r \geq 2$,

$$\mu^{r+1}(G) \leq (r+1)k_{r+1}(G) + \sum_{s=2}^r (s-1)k_s(G)\mu^{r+1-s}(G),$$

and, if G is of order n , then

$$k_{r+1}(G) \geq \left(\frac{\mu(G)}{n} - 1 + \frac{1}{r} \right) \frac{r(r-1)}{r+1} \left(\frac{n}{r} \right)^{r+1}.$$

Keywords: *number of cliques, clique number, spectral radius, stability*

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