# Eigenvalues and forbidden subgraphs I

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**Abstract**

Suppose a graph $G$ have $n$ vertices, $m$ edges, and $t$ triangles. Letting $\lambda_n(G)$ be the largest eigenvalue of the Laplacian of $G$ and $\mu_n(G)$ be the smallest eigenvalue of its adjacency matrix, we prove that

$$\lambda_n(G) \geq \frac{2m^2 - 3nt}{m(n^2 - 2m)},$$
$$\mu_n(G) \leq \frac{3n^3t - 4m^3}{nm(n^2 - 2m)},$$

with equality if and only if $G$ is a regular complete multipartite graph.

Moreover, if $G$ is $K_{r+1}$-free, then

$$\lambda_n(G) \geq \frac{2mn}{(r-1)(n^2 - 2m)}$$

with equality if and only if $G$ is a regular complete $r$-partite graph.

**Keywords:** $K_r$-free graph, graph Laplacian, largest eigenvalue, smallest eigenvalue, forbidden subgraphs