

# The Generalised Randić Index of Trees

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## Abstract

The Generalised Randić index  $R_{-\alpha}(T)$  of a tree  $T$  is the sum over the edges  $uv$  of  $T$  of  $(d(u)d(v))^{-\alpha}$  where  $d(x)$  is the degree of the vertex  $x$  in  $T$ . For all  $\alpha > 0$ , we find the minimal constant  $\beta_c = \beta_c(\alpha)$  such that for all trees on at least 3 vertices  $R_{-\alpha}(T) \leq \beta_c(n+1)$  where  $n = |V(T)|$  is the number of vertices of  $T$ . For example, when  $\alpha = 1$ ,  $\beta_c = \frac{15}{56}$ . This bound is sharp up to the additive constant — for infinitely many  $n$  we give examples of trees  $T$  on  $n$  vertices with  $R_{-\alpha}(T) \geq \beta_c(n-1)$ . More generally, fix  $\gamma > 0$  and define  $\tilde{n} = (n - n_1) + \gamma n_1$ , where  $n = n(T)$  is the number of vertices of  $T$  and  $n_1 = n_1(T)$  is the number of leaves of  $T$ . We determine the best constant  $\beta_c = \beta_c(\alpha, \gamma)$  such that for all trees on at least 3 vertices,  $R_{-\alpha}(T) \leq \beta_c(\tilde{n}+1)$ . Using these results one can determine (up to  $o(n)$  terms) the maximal Randić index of a tree with a specified number of vertices and leaves. Our methods also yield bounds when the maximum degree of the tree is restricted.

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