Connectivity of addable graph classes

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A non-empty class $\mathcal{A}$ of labelled graphs that is closed under isomorphism is weakly addable if for each graph $G \in \mathcal{A}$ and any two distinct components of G, any graph that can be obtain by adding an edge between the two components is also in $\mathcal{A}$. For a weakly addable graph class $\mathcal{A}$, we consider a random element R_n chosen uniformly from the set of all graph in $\mathcal{A}$ on the vertex set $\{1, \ldots, n\}$. McDiarmid, Steger and Welsh conjecture [3] that the probability that R_n is connected is at least $e^{-1/2} + o(1)$ and showed that it is at least e^{-1} for sufficiently large n. We improve the latter result and show that this probability is at least $e^{-0.7983}$ for sufficiently large n. We also consider 2-addable graph classes $\mathcal{B}$ where for each graph $G \in \mathcal{B}$ and for any two distinct components of G, the graphs that can be obtained by adding at most 2 edges between the components are in $\mathcal{B}$. We show that a random element of a 2-addable graph class on n vertices is connected with probability tending to 1 as n tends to infinity.

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