# An efficient $\gamma$-model BGK scheme for multicomponent flows on unstructured meshes

Song Jiang Guoxi Ni*
LCP, Institute of Applied Physics and Computational Mathematics
P.O. Box 8009, Beijing 100088, China.
E-mail: jiang@iapcm.ac.cn, gxni@iapcm.ac.cn

**Abstract**

Based on the $\gamma$-model, the BGK scheme and the time-dependent gas distribution function, we present an efficient second-order gas-kinetic BGK scheme for compressible multicomponent flows on unstructured meshes. The second-order accuracy of the scheme is achieved by including slopes in the reconstruction step and the efficiency lies in the fact that the slopes in a cell are constructed by using the information in the cell only. Using the explicitly constructed time-dependent gas distribution function, we can obtain in addition the values of the flow variables at cell edges of the cell, which together with the cell averaged values make it possible to construct the slopes of the flow variables in the cell, avoiding the use of the values from neighboring cells in the usual process. Thus, with the stencil of a first-order scheme, we construct a second-order gas-kinetic scheme. Numerical examples illustrate the accuracy and efficiency of the scheme.

Classification (2000): 76M25, 65M99

Keywords: Multicomponent flows, efficient 2nd-order BGK scheme, $\gamma$-model, unstructured meshes

## 1 Introduction

In recent years, the development of Boltzmann-type schemes has attracted much attention. The success of such schemes has appeared in a wide range of applications, see [13, 14, 6, 16] for example. Among the Boltzmann-type schemes, the equilibrium-flux method (EFM) has been intensively studied [18]. EFM is flux splitting and is also referred to as a kinetic flux vector splitting (KFVS) scheme. With the inclusion of a Boltzmann collision model, the BGK model, in the flux evaluation process, the gas-kinetic BGK scheme has been proposed in [17, 24] (also see the survey article [25]). The BGK scheme differs from the KFVS method mainly in the inclusion of particle collisions in the gas evolution stage. Instead of solving the collisionless Boltzmann equation, the BGK scheme uses a collisional BGK model which approximates the Boltzmann collision operator when it is near equilibrium. Since the gas evolution process is associated with a relaxation process, i.e., from a non-equilibrium state to an equilibrium one, the entropy condition is satisfied by the BGK scheme. Once the physical structure can be well resolved by the numerical cell size, the BGK scheme automatically gives an accurate compressible Navier-Stokes solution in smooth regions, while in the discontinuous regions, the delicate dissipative mechanism in the BGK scheme generates a stable and crisp shock transition.

Numerical schemes for multicomponent flows associated with discontinuities and shock waves have been one of the most important topics in computational fluid dynamics. In the last decades great progress has been made, and a number of schemes have been proposed in the literature. Among them are methods which use an extended conservative system of governing equations, where additional conservation

*Corresponding author