Multiple peak solutions for Bose-Einstein condensates in a periodic potential

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Abstract

We study multiple peak solutions of Bose-Einstein condensates (BEC) in a periodic potential. A staircase multigrid-continuation algorithm is presented for computing energy levels of the BEC. The proposed algorithm is a modification of the two-grid discretization schemes [6] or the simplified two-grid schemes [22] for tracing solution curves of semilinear elliptic eigenvalue problems. The algorithm has the following advantages over the two-grid or the simplified two-grid schemes: (i) It guarantees that the scheme will converge to the target point on the finest grid. (ii) It is cheaper than the simplified two-grid scheme. We apply the staircase-multigrid continuation algorithm to study the ground-state and the first-few excited solutions of the 1D, 2D and 3D BEC in a periodic potential. Our numerical results show that if the chemical potential is large enough, the number of peaks for the ground-state solutions is $\prod_{j=1}^{n}(\frac{1}{d_j}-1)$, where d_1 is the distance of neighbor wells in the x-coordinate, and so on, and n the dimension of the BEC. Moreover, the number of peaks is less than $\prod_{j=1}^{n}(\frac{1}{d_j}-1)$ for the excited-state solutions. Additionally, the global wave number of the ground-state and the excited-state solutions is the same as that of the associated linear eigenvalue problem if d_i is/are small enough. The numerical results are consistent with the mathematical formulation and the theoretical prediction of the BEC in a periodic potential.

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