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Spectral Relationships Between Kicked Harper and On–Resonance Double Kicked Rotor Operators

Wayne Lawton*

Department of Mathematics, National University of Singapore, 2 Science Drive 2, Singapore 117543

Anders S. Mouritzen†

Department of Physics and Center of Computational Science and Engineering, National University of Singapore, 117542, Singapore and Department of Physics and Astronomy, University of Aarhus, DK-8000, Aarhus C, Denmark

Jiao Wang

Temasek Laboratories, National University of Singapore, 117542, Singapore and Beijing-Hong Kong-Singapore Joint Center for Nonlinear and Complex Systems (Singapore), National University of Singapore, 117542, Singapore

Jiangbin Gong

Department of Physics and Center of Computational Science and Engineering, National University of Singapore, 117542, Singapore and NUS Graduate School for Integrative Sciences and Engineering, Singapore 117597, Singapore

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Kicked Harper operators and on–resonance double kicked rotor operators model quantum systems whose semiclassical limits exhibit chaotic dynamics. Recent computational studies indicate a striking resemblance between the spectrums of these operators. In this paper we apply C^*–algebra methods to explain this resemblance. We show that each pair of corresponding operators belong to a common rotation C^*–algebra $\mathcal{B}_\alpha$, prove that their spectrums are equal if α is irrational, and prove that the Hausdorff distance between their spectrums converges to zero as q increases if $\alpha = p/q$ with p and q coprime integers. Moreover, we show that corresponding operators in $\mathcal{B}_\alpha$ are homomorphic images of mother operators in the universal rotation C^*–algebra $\mathcal{A}_\alpha$ that are unitarily equivalent and hence have identical spectrums. These results extend analogous results for almost Mathieu operators. We also utilize the C^*–algebraic framework to develop efficient algorithms to compute the spectrums of these mother operators for rational α and present preliminary numerical results that support the conjecture that their spectrums are Cantor sets if α is irrational. This conjecture for almost Mathieu operators, called the Ten Martini Problem, was recently proved after intensive efforts over several decades. This proof for the almost Mathieu operators utilized transfer matrix methods, which do not exist for the kicked operators. We outline a strategy, based on a special property of loop groups of semisimple Lie groups, to prove this conjecture for the kicked operators.

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*Electronic address: wlawton@math.nus.edu.sg

†Electronic address: asm@phys.au.dk