UNIQUENESS OF ROOTS UP TO CONJUGACY FOR SOME AFFINE AND FINITE TYPE ARTIN GROUPS

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Abstract. Let G be one of the Artin groups of finite type $\mathbf{B}_n = \mathbf{C}_n$ and affine type $\tilde{\mathbf{A}}_{n-1}$, $\tilde{\mathbf{C}}_{n-1}$. In this paper, we show that if α and β are elements of G such that $\alpha^k = \beta^k$ for some nonzero integer k, then α and β are conjugate in G. For the Artin group of type $\mathbf{A}_n$, this was recently proved by J. González-Meneses.

In fact, we prove a stronger theorem, from which the above result follows easily by using descriptions of those Artin groups as subgroups of the braid group on $n+1$ strands. Let P be a subset of $\{1, \ldots, n\}$. An n-braid is said to be *P-pure* if its induced permutation fixes each $i \in P$, and *P-straight* if it is P-pure and it becomes trivial when we delete all the i-th strands for $i \notin P$. Exploiting the Nielsen-Thurston classification of braids, we show that if α and β are P-pure n-braids such that $\alpha^k = \beta^k$ for some nonzero integer k, then there exists a P-straight n-braid γ with $\beta = \gamma\alpha\gamma^{-1}$. Moreover, if $1 \in P$, the conjugating element γ can be chosen to have the first strand algebraically unlinked with the other strands. Especially in case of $P = \{1, \ldots, n\}$, our result implies the uniqueness of roots of pure braids, which was known by V. G. Bardakov and by D. Kim and D. Rolfsen.

Keywords: Artin group, braid group, uniqueness of roots
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