WHEN IS AN L-FUNCTION NON-VANISHING IN PART OF THE CRITICAL STRIP?

STEPHEN GELBART

DEDICATED TO ROGER HOWE ON HIS 60TH BIRTHDAY -
WITH FRIENDSHIP AND ADMIRATION

Abstract

In this paper we discuss three kinds of methods for proving that an L-function is non-zero in a part of its critical strip. The fundamental classical result is de la Vallée Poussin's theorem with Riemann's zeta function. A first method of generalizing this fits in quite nicely to the Rankin-Selberg theory of L-functions for $GL(n)$. It falls short, however, of proving all cases of our Main Theorem. A second method of proof using Eisenstein series for $SL(2)$ was introduced by Sarnak following Selberg. This was carried out in 2002 [Sar]. In his paper, Sarnak suggested generalizing his method using Langlands-Shahidi theory. The result was [GLS] and [Gel-Lap]. This third method works not only for the $GL(n)$ L-functions but for a broad range of Langlands-Shahidi L-functions too. For a result by this method which goes beyond "de la Vallee Poussin's method" see the example of the ninth symmetric L-function of $GL(2)$.