

The inversion formula and holomorphic extension of the minimal representation of the conformal group

Toshiyuki KOBAYASHI* and Gen MANO

RIMS, Kyoto University,
Sakyo-ku, Kyoto, 606-8502, Japan

toshi@kurims.kyoto-u.ac.jp; gmano@kurims.kyoto-u.ac.jp

Dedicated to Roger Howe on the occasion of his 60th birthday

Abstract

The minimal representation π of the indefinite orthogonal group $O(m+1, 2)$ is realized on the Hilbert space of square integrable functions on \mathbb{R}^m with respect to the measure $|x|^{-1} dx_1 \cdots dx_m$. This article gives an explicit integral formula for the holomorphic extension of π to a holomorphic semigroup of $O(m+3, \mathbb{C})$ by means of the Bessel function. Taking its ‘boundary value’, we also find the integral kernel of the ‘inversion operator’ corresponding to the inversion element on the Minkowski space $\mathbb{R}^{m,1}$.

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