SMALL SEMISIMPLE SUBALGEBRAS OF SEMISIMPLE LIE ALGEBRAS

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To Roger Howe, with friendship and admiration.

ABSTRACT. Let $\mathfrak{g}$ denote a semisimple Lie algebra with the property that none of its simple factors is of type A_1. Suppose that $\mathfrak{k} \subseteq \mathfrak{g}$ is a Lie subalgebra isomorphic to $\mathfrak{sl}_2$. The goal of this paper is to prove the existence of a positive integer $b(\mathfrak{k}, \mathfrak{g})$ such that if V is any irreducible finite dimensional $\mathfrak{g}$-module then when restricted to $\mathfrak{k}$, the decomposition of V will contain some irreducible $\mathfrak{k}$-module with dimension less than $b(\mathfrak{k}, \mathfrak{g})$. Beyond proving the theorem, we show how it may be generalized by introducing the notion of a *small* subalgebra.