INTRADAY DIVERSIFIED WORLD STOCK INDICES:
DYNAMICS, RETURN DISTRIBUTIONS, DEPENDENCE STRUCTURE

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OUTLINE

Introduction

I. Construction of world stock indices

II. A universal world index dynamics

III. Empirical results
   a) Intraday seasonality
   b) Return distributions
   c) Copulas

IV. Conclusion
INTRODUCTION

- Motivation
- The Philosophy
- Applications
MOTIVATION

• We have
  – empirical knowledge on high-frequency financial data and
  – a theory: the benchmark model (growth optimal portfolio (GOP) in a multivariate stochastic volatility framework).

• The idea:
  Making the best use of both of them (Synergy).
THE PHILOSOPHY

• To create a portfolio with universal properties from available high-frequency data,

• to show that it approximates the GOP (market portfolio), and

• to analyse this portfolio theoretically and empirically.
APPLICATIONS

• Benchmark model excludes arbitrage without requiring the existence of an equivalent risk neutral martingale measure.

• An index approximating the GOP will serve as basis for:
  – Short term option pricing
  – Hedging
  – Portfolio management
  – Risk measurement
I. CONSTRUCTION OF WORLD STOCK INDICES
CONSTRUCTION OF WORLD CAPITAL INDICES

- Constructed as self-financing portfolio of local stock market indices.
- Coverage: 34 stock indices throughout the world. Corresponds to between 2000 and 3000 stocks.
- Use accumulation indices $I^{(j,c)}(t)$.
- Transformation of price indices into accumulation indices:

$$I^{(j,c)}(t) = P^{(j,c)}(t) \exp \left\{ \int_0^t y^{(j)}(u) du \right\}$$

where $P^{(j,c)}(t)$ is the local stock spot price index, $t \in [0,T]$, $j \in \{1, 2, \ldots, d\}$, $c \in \{\text{USD, CHF, } \ldots\}$, and $y^{(j)}(t)$ the continuously compounded $j$th dividend yield.
CONSTRUCTION OF WSIs: THE DATA

- High frequency tick-by-tick data from April 1996 till June 2001 for
  - stock indices of 34 countries and
  - the corresponding FX rates.
  - Previous-tick interpolation to regularly spaced 5 min. data

- Daily data for
  - local dividend yields
  - USD interest rates

- High frequency data provided by Olsen Data.

- Daily data provided by Thomson Financial.
CONSTRUCTION OF WSIs: COUNTRIES

Africa:
- South Africa

America:
- Argentinia
- Brasil
- Canada
- Mexico
- United States

Asia-Pacific:
- Australia
- Hong Kong
- India
- Indonesia
- Japan
- Korea
- Malaysia

Europe:
- Austria
- Belgium
- Denmark
- Finland
- France
- Germany
- Greece
- Hungary
- Ireland
- Italy
- Netherlands
- Norway
- Portugal
- Spain
- Sweden
- Switzerland
- Turkey
- UK
CONSTRUCTION OF WSIs (CONT.)

- All indices should be denominated in USD.
- Change of denomination:
  \[ I^{(j, \text{USD})}(t) = \frac{I^{(j, c)}(t)}{FX^{\text{USD}/c}(t)} \]
  with the foreign exchange rate \( FX^{\text{USD}/c}(t) \) at time \( t \).
- WSI \( V^{(\text{WSI})}(t) \) denominated in USD is a portfolio defined as
  \[ V^{(\text{WSI})}(t) = \sum_{j=1}^{d} \delta^{(j)}_{(\text{WSI})}(t) I^{(j, \text{USD})}(t). \]
- Strategy \( \delta^{(j)}_{(\text{WSI})}(t) \) with proportions
  \[ \pi^{(j)}_{(\text{WSI})}(t) = \delta^{(j)}_{(\text{WSI})}(t) \frac{I^{(j, \text{USD})}(t)}{V^{(\text{WSI})}(t)}. \]
CONSTRUCTION OF WSIs: STRATEGIES

We consider three WSIs:

- *Equal Weighted Index* (EWI):
  Developed markets get weight 1, and emerging markets weight 0.5 (up to normalisation).

- *Market Capitalisation Index* (MCI):
  Proportions $\pi_{(\text{WSI})}^{(j)}(t)$ correspond to market capitalisation. Index based on more than 95% of total world market capitalisation.

- *Gross Domestic Product Index* (GDPI):
  Proportions $\pi_{(\text{WSI})}^{(j)}(t)$ correspond to gross domestic product. Index based on more than 85% of the total world GDP.
MARKET CAPITALISATION BASED PROPORTIONS

Proportions


Econometric Forecasting and High Frequency Data

NUS – IMS
CONSTRUCTION OF WSIs: REBALANCING

The WSIs are rebalanced

- either after one year has elapsed
- or when an index is to be added to the WSI, whichever occurs first.

Rebalancing dates:

CONSTRUCTION OF WSIs: DISCOUNTING AND NORMALISATION

- **Discounted WSI** $\tilde{V}^{(\text{WSI})}(t)$ at time $t$:
  \[
  \tilde{V}^{(\text{WSI})}(t) = V^{(\text{WSI})}(t) \exp \left\{ - \int_0^t r(u)du \right\}
  \]
  for $t \in [0, T]$, where
  $r(t)$ is the *short term interest rate* for the US market.

- **Normalised WSI**:
  \[
  Y^{(\text{WSI})}(t) = \frac{\tilde{V}^{(\text{WSI})}(t)}{\bar{\alpha}(t)}
  \]
  with $\bar{\alpha}(t) = \alpha_0 \exp(\eta t)$, $\eta = 0.048$ average growth rate.
HIGH FREQ. WORLD STOCK MARKET INDICES
\[ Y^{(MSCl)}(t) = S^{(MSCl)} / \bar{\alpha}(t), \quad \bar{\alpha}(t) = \alpha_0 \exp(\eta t), \quad \eta = 0.048. \]
HOURLY RETURNS FOR MARKET-CAP. BASED INDEX
II. UNIVERSAL INDEX DYNAMICS
UNIVERSAL INDEX DYNAMICS: COOKING RECIPE

- Take a multivariate stochastic volatility framework with
- \( n \) (accumulated) risky instruments

\[
dS^{(j)}(t) = S^{(j)}(t) \left\{ a^j(t) \, dt + \sum_{k=1}^{d} b^{j,k}(t) \, dW^k(t), \right\}
\]

- 1 riskless instrument (deterministic drift, no volatility), and
- a self-financing portfolios \( S' \).
- Compute the mean-variance optimisation of \( S' \).
- Formulated everything thoroughly in the language of stochastic processes.
- Turn the handle.
UNIVERSAL INDEX DYNAMICS: RESULT

- The growth optimal portfolio (GOP):

\[
dV^{(\delta_*)}(t) = V^{(\delta_*)}(t) \left[ r(t) \, dt + \sum_{k=1}^{d} \theta^k(t) \left( \theta^k(t) \, dt + dW^k(t) \right) \right]
\]

with market price of risk

\[
\theta(t) = (\theta^1(t), \theta^2(t), \ldots, \theta^d(t))^\top
\]

\[
= b^{-1}(t) \left[ a(t) - r(t) \mathbf{1} \right]
\]

and riskless interest rate \( r(t) \).
DYNAMICS: DISCOUNTED GOP

- Discounted GOP:
  \[ \bar{V}^{(\delta_*)}(t) = \frac{V^{(\delta_*)}(t)}{V^{(0)}(t)} \]

  with riskless instrument
  
  \[ V^{(0)}(t) = \exp \left\{ \int_0^t r(s) ds \right\} . \]

  is squared Bessel process of dimension 4.

  \[ \rightarrow \] Returns \( t \)-distributed with 4 degrees of freedom.
DYNAMICS: DISCOUNTED GOP (CONT.)

- Dynamics:

\[
d\tilde{V}^{(\delta_*)}(t) = \tilde{V}^{(\delta_*)}(t) \|\theta(t)\| \left( \|\theta(t)\| dt + d\hat{W}(t) \right)
\]

with

- standard Wiener process

\[
d\hat{W}(t) = \frac{1}{\|\theta(t)\|} \sum_{k=1}^{d} \theta^k(t) dW^k(t)
\]

- diffusion constant

\[
\|\theta(t)\| = \left[ \sum_{k=1}^{d} (\theta^k(t))^2 \right]^{1/2}
\]

- drift

\[
\alpha(t) = \tilde{V}^{(\delta_*)}(t) \|\theta(t)\|^2
\]

- market price of risk \( \|\theta(t)\| \).

- Drift and Volatility are related, similar to CAPM, but with endogenously determined market price of risk.
DYNAMICS: DRIFT PROCESS AND MARKET ACTIVITY

- Modelling of $\alpha(t)$ is essential.

- Drift $\alpha(t)$ depends on market activity $m(t)$:

$$\alpha(t) = \xi \exp \left\{ \eta \int_0^t m(s) \, ds \right\} m(t).$$

Important for intra-day analysis.

- Market activity gives rise to market activity time:

$$\psi(t) = \psi(t_0) + \int_{t_0}^t m(s) \, ds.$$
DYNAMICS: NORMALISED GOP

- Discounting normalised GOP by the drift: \( Y(t) = \tilde{V}^{(\delta_\ast)}(t)/\alpha(t) \)

  with \( dY(t) = \eta \left( \frac{1}{\eta} - Y(t) \right) m(t) \, dt + \sqrt{Y(t) \, m(t)} \, d\hat{W}(t). \)

- Transformation to market activity time, \( Y_\psi(t) = Y(t) \):

  \[ dY_\psi = \eta \left( \frac{1}{\eta} - Y_\psi \right) \, d\psi + \sqrt{Y_\psi} \, d\hat{W}_\psi \]

  with \( d\hat{W}_\psi = \sqrt{m(t)} \, d\hat{W}(t). \)

- This is a square root process with dimension 4.

  \( \eta \) is the only constant.
DYNAMICS: THE SQUARE ROOT PROCESS

- Square root process:
  \[ d \left( \sqrt{Y_\psi} \right) = \left( \frac{3}{8 \sqrt{Y_\psi}} - \frac{\eta}{2 \sqrt{Y_\psi}} \right) d\psi + \frac{1}{2} d\hat{W}_\psi. \]

- Quadratic variation:
  \[ \langle \sqrt{Y} \rangle_\psi = \frac{\psi}{4}. \]

- Market activity time can easily determined empirically by:
  \[ \psi(t) = 4 \langle \sqrt{Y} \rangle_t. \]
III. EMPIRICAL RESULTS
WHY DO WORLD INDICES APPROXIMATE THE GOP?

- Limit theorem (Platen, 2003):

  Every well-diversified portfolio approaches the GOP in the limit of infinite number of instruments.

  → **All world indices should look similar.**
APPLICATION TO WORLD INDICES

- Discounted MSCI: \( S^{(MSCI)}(t) \).

- Normalised MSCI:
  \[
  Y^{(MSCI)}(t) = \bar{S}^{(MSCI)}/\bar{\alpha}(t)
  \]
  with \( \bar{\alpha}(t) = \alpha_0 \exp(\eta t) \),
  \( \eta = 0.048 \) average economic growth rate.

- The square root process:
  \[
  \sqrt{Y^{(MSCI)}(t)}.
  \]

- Quadratic variation of the square root process:
  \[
  \left\langle \sqrt{Y^{(MSCI)}} \right\rangle_t.
  \]
QUADRATIC VARIATION OF A DAILY INDEX

Quadratic variation $\left\langle \sqrt{Y^{(MSCI)}} \right\rangle_t$
EMPIRICAL RESULTS:
INTRADAY SEASONALITY
QUADRATIC VARIATION FOR AN INTRADAY INDEX
LOGARITHM OF MARKET ACTIVITY, $\log(m(t))$
MODEL FOR MARKET ACTIVITY

- Multiplicative noise with seasonal volatility.
- Mean-reverting stochastic process:

\[ dm(t) = \frac{\gamma}{2} \beta^2(t) \left( \frac{p(t) - 1}{\gamma} - m(t) \right) \, dt + \beta(t) m(t) \, dW(t). \]

where
- \( p(t) > 0 \) is the reference market activity
- \( \beta(t) > 0 \) is the activity volatility.
NORM. MARKET ACTIVITY: QUAD. VARIATION
STATIONARY DENSITY OF MARKET ACTIVITY

- Market activity in activity volatility time has stationary density ∼ gamma density:
  \[ \bar{p}_t(l; \kappa) = \frac{(\kappa)^{\kappa \bar{m}}}{\Gamma(\kappa \bar{m})} \exp \left\{ -\kappa e^l \right\} e^{l(\kappa \bar{m} - 1)}. \]

- To suppress unwanted spikes we use a “restricted” log-likelihood function to estimate \( \kappa \)
  \[ L(\kappa) = \sum_{n=1}^{n_t^f} 1_{\{\tau_n > \underline{l}\}} \ln \bar{p}_t(l_{\tau_n}; \kappa) \]
  with \( \underline{l} = -0.2. \)
OBSERVED AND ESTIMATED STATIONARY DENSITY
**RELAXATION OF MARKET ACTIVITY**

Estimation of the relaxation of market activity to its average value:

<table>
<thead>
<tr>
<th>Index</th>
<th>$\hat{\gamma}$ [year$^{-1}$]</th>
<th>$\hat{p}$ [year$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCI</td>
<td>103.2</td>
<td>105.8</td>
</tr>
<tr>
<td></td>
<td>(89.3,116.4)</td>
<td>(92.3,119.2)</td>
</tr>
<tr>
<td>GDPI</td>
<td>136.6</td>
<td>138.9</td>
</tr>
<tr>
<td></td>
<td>(120.7,152.5)</td>
<td>(122.5,155.3)</td>
</tr>
<tr>
<td>EWI</td>
<td>137.3</td>
<td>139.7</td>
</tr>
<tr>
<td></td>
<td>(121.2,153.4)</td>
<td>(123.5,155.8)</td>
</tr>
</tbody>
</table>
DYNAMICS OF NORMALISED WSIs

- Normalised GOP is square root process of dimension 4.
- Market activity time proportional to quadratic variation of square root process:
  \[ \psi(t) = 4 \left\langle \sqrt{Y} \right\rangle_t \]
- Results of linear regression:

<table>
<thead>
<tr>
<th>Index</th>
<th>Slope Coefficient</th>
<th>( R^2 )</th>
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<tbody>
<tr>
<td>MCI</td>
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<tr>
<td>GDPI</td>
<td>0.232</td>
<td>0.9969</td>
</tr>
<tr>
<td>EWI</td>
<td>0.198</td>
<td>0.9919</td>
</tr>
</tbody>
</table>
EMPIRICAL RESULTS:
RETURN DISTRIBUTIONS
CLASSES OF DISTRIBUTIONS

Symmetric generalized hyperbolic density:

\[ f_y(x) = \frac{1}{\delta \sqrt{\Delta}} K_\lambda(\bar{\alpha}) \sqrt{\frac{\bar{\alpha}}{2\pi}} \left( 1 + \frac{(x - \mu \Delta)^2}{\delta^2 \Delta} \right)^{\frac{\lambda}{2} - \frac{1}{2}} K_{\lambda - \frac{1}{2}} \left( \bar{\alpha} \sqrt{1 + \frac{(x - \mu \Delta)^2}{\delta^2 \Delta}} \right) \]

Student-\( t \) density:

\[ f_y(x) = \frac{\Gamma\left(\frac{1}{2} \nu + \frac{1}{2}\right)}{\varepsilon \sqrt{\pi \nu \Delta} \Gamma\left(\frac{1}{2} \nu\right)} \left( 1 + \frac{(x - \mu \Delta)^2}{\varepsilon^2 \nu \Delta} \right)^{-\frac{1}{2} \nu - \frac{1}{2}} \]

Normal-inverse Gaussian density:

\[ f_y(x) = \frac{\sqrt{\bar{\alpha}} \exp\{\bar{\alpha}\}}{c \sqrt{\Delta} \pi} \left( 1 + \frac{(x - \mu \Delta)^2}{\bar{\alpha} c^2 \Delta} \right)^{-\frac{1}{2}} K_1 \left( \bar{\alpha} \sqrt{1 + \frac{(x - \mu \Delta)^2}{\bar{\alpha} c^2 \Delta}} \right) \]
HYPERBOLIC DENSITY:

\[ f_y(x) = \frac{1}{2 \delta \sqrt{\Delta} K_1(\bar{\alpha})} \exp \left\{ -\bar{\alpha} \sqrt{1 + \frac{(x - \mu)^2}{\delta^2 \Delta}} \right\} \]

VARIANCE-GAMMA DENSITY:

\[ f_y(x) = \frac{\sqrt{\lambda}}{c \sqrt{\Delta} \pi \Gamma(\lambda) 2^{\lambda - 1}} \left( \sqrt{2 \lambda} \frac{|x - \mu \Delta|}{c \sqrt{\Delta}} \right)^{\lambda - \frac{1}{2}} K_{\lambda - \frac{1}{2}} \left( \sqrt{2 \lambda} \frac{|x - \mu \Delta|}{c \sqrt{\Delta}} \right) \]
EXPECTED RESULTS

- Intraday time horizon:
  - Variance $\sim$ market activity is Gamma-distributed.
    $\rightarrow$ Variance-gamma distributed returns.

- Daily time horizon:
  - No appreciable seasonalities.
    $\rightarrow 4 \langle \sqrt{Y} \rangle_t \simeq t$
    $\rightarrow t$-4 distributed returns.
<table>
<thead>
<tr>
<th>Currency</th>
<th>$\bar{\alpha}$</th>
<th>$\lambda$</th>
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</thead>
<tbody>
<tr>
<td>ARS</td>
<td>0.000</td>
<td>-1.574</td>
</tr>
<tr>
<td>ATS</td>
<td>0.560</td>
<td>-1.626</td>
</tr>
<tr>
<td>AUD</td>
<td>0.000</td>
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<tr>
<td>BEF</td>
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<td>DKK</td>
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<td>ESP</td>
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</tr>
<tr>
<td>FRF</td>
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<tr>
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<tr>
<td>USD</td>
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<tr>
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<tr>
<td>0.000</td>
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<td>0.000</td>
</tr>
<tr>
<td>-1.574</td>
<td>-2.044</td>
<td>-1.428</td>
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AGGREGATED DAILY DISTRIBUTIONS

<table>
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<tr>
<th>Dist.</th>
<th>$\bar{\alpha}$</th>
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<th>loglikelihood</th>
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<tbody>
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<td>sgh</td>
<td>$2.21 \cdot 10^{-6}$</td>
<td>-1.8003</td>
<td>-388187.445374</td>
</tr>
<tr>
<td>t</td>
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<td>-1.8003</td>
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<tr>
<td>nig</td>
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</table>

According to likelihood ratio test, log. returns are $t$-distributed with 0.1% significance level.
## HOURLY DISTRIBUTIONS

<table>
<thead>
<tr>
<th>Currency</th>
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<th>$\lambda$</th>
<th>Loglikelihood</th>
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</thead>
<tbody>
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EMPIRICAL RESULTS:
DEPENDENCE STRUCTURE
Akaike Information Criteria for different time horizons

The copula has been computed for the GOP denominated in USD and in CHF.

The copula has been fitted to the MCI.
CONCLUSION

- Intraday world stock indices (WSIs) constructed.
- WSIs can be approximated by growth optimal portfolio and displays universal dynamic behavior.
- Market-capitalisation based WSI (MCI) has maximum growth and therefore approximates best the GOP.
- Intraday seasonal market activity incorporated.
- Normalised WSIs are square root processes of dimension 4 in market activity time.
CONCLUSION (CONT.)

- Return distribution of WSI:
  - $t$-distributed with approx. 4 degrees of freedom for daily time horizons.
  - Variance-gamma distributed for intraday time horizons.

- For the modelling of the dependence structure, $t$-copula provides best fit out of a set of standard copulas.
CONCLUSION: OPEN QUESTIONS

There are many!