Moments of Fat-Tailed Distributions

\[ \int_{-\infty}^{\infty} \phi(x) \, dx = \mu \]

\[ \int_{-\infty}^{\infty} \phi(x) \, dx = \mu \]

\[ \int_{-\infty}^{\infty} \phi(x) \, dx = \mu \]

Characterization of Fat-Tailed Distributions

Classification of Distributions

Classification of Distributions

Financial Distributions

How Heavy Are Tails Of

Bounded distributions without tails

decays as a power with exponent \( \alpha \)

Cumulative distribution function asymptotically

Fat-tailed distributions

All moments are finite

exponentially in the tails

Cumulative distribution function decays

Thin-tailed distributions
HILL ESTIMATOR: PROPERTIES

- No easy way to determine base value of \( \lambda \)
- \( \lambda \) depends on \( u \)
- \( \lambda \) based for finite sample size
- \( \lambda \) asymptotically normally distributed

**Empirical Estimation of Tail Behavior**

Let \( X_1, X_2, \ldots, X_n \) be a sample of \( n \) independent observations

\[
\sum_{i=1}^{n-1} I_{\{X_i < \lambda - \frac{u}{i} \}}
\]

The so-called HILL estimator (Hill, 1975)

with unknown probability distribution function \( F \)

**Empirical Results**

- Tail index computed separately for every modified sample
- One-tenth of the total sample
- Data sample modified in 10 different ways by removing
- Computation of standard error by jackknife method
- Approximate 95% confidence corresponding to \( \lambda \) distributed
- Confidence ranges are standard errors times 1.96
- Estimated by a subsample bootstrap method

**Tail Behavior of Student-t Distribution**

Sample size: 10,000. Average computed over 50 simulations.
**SUMMARY**

- Aluminum against the capability of systems like the EMS.
- Reduced volatility induced by the EMS rules at the cost of a
- EMS acts to lower tail indices around 2.7.
- (Structural change) volatility indicating the EMS and much lower volatility in the G08.
- These markets differ from the FX market with very high.
- Gold and silver have tail indices above 4.
- (and 3.9)
- For 30min returns all indices are above 3.5 (between 3.1).
- For 30min returns all indices are above 3.5 (between 3.1).

**RESULTS**

- High adjustment to external shocks.
- Low degree of regulation.
- Tail Index small.
- Structural adjustment to external shocks.
- The interactions between agents with different time horizons.
- They are an empirical measure of market regulation and efficiency.
- Tail Index large.
- Tail Index reflects.

---

**TAIL INDICES FOR MAIN FX RATES**

---

**TAIL INDICES FOR SOME CROSS RATES**

---
Aggregation of $t$-3.5 distribution

Textual content:

- Volatility autocorrelation
  - Absolute returns are preferred for computation of autocorrelations
  - High moments of return distributions usually decline
  - And moments of return distributions finite
  - Tail indices between 2 and 4

RESULTS: SUMMARY (CONT.)

- Extreme risks in financial markets
  - What is the best hedging strategy?
  - Are there theoretical processes to model them?
  - Where are the extreme movements to be expected?

Detailed studies have shown that 18 years of daily data are not sufficient to estimate the tail index reliably.

- Sample size decreases
  - (Relative) returns
  - Transmission to tail behavior occurs at increasingly larger
  - Because:
    - For larger time horizons tail behavior can no longer be observed
  - Therefore non-stable

Return distributions are fat-tailed with tail index $> 2$ and
Volatility patterns
Intraday and Intra-week

Returns (after deseasonalization)
- Slowly decaying autocorrelation function of absolute returns
- Autocorrelation function of absolute returns pronounced daily and weekly seasonally in

and Seasonalities
S.E. Volatility Autocorrelation

Extreme cases:
- Localized exchange-traded futures
- OTC markets
- FX spot market

The patterns may change when the institutional setting is changed (example: change of the operating hours of a market).
LONG RANGE VOLATILITY AUTOCORRELATIONS

After deseasonalization:

Autocorrelation function of absolute returns displays

ACF OF DESEASONALIZED Hourly ABS. RETURNS

EXCHANGE-TRADED INSTRUMENTS

SEASONAL VOLATILITY PATTERNS FOR DAX

CHANGE OF DAILY ACTIVITY PERIOD FOR DAX
**Properties:**

- Fractal appearance of scaling.
- Empirical scaling behavior of higher moments.
- An application of scaling behavior analysis.
- Empirical scaling behavior of first moments of returns.

**S.F.: SCALING LAWS**

---

The drift exponent reflects the institutional framework of FX rates. The drift exponent affects the beta on the right (ITJ index and FRP) and (FRP) on the left (DEN index and FRP the standard) and (a) on the left (DEN index and FRP) and (b) on the right (DEN index and FRP) increases the drift exponent of the scaling law for USD rates.

**Dependency of mean absolute return on the size of the time interval**

A Gaussian i.i.d. random walk would have $D = 0.5$. With a confidence of specifically $p < 0.05$ for the drift exponent:

$|D| < 0.5$ implies the normal size and $|D| > 0.5$ implies a constant.

Empirically we find a power law for mean absolute returns:

$D = \gamma \cdot \sqrt{\text{time interval}}$
The scaling exponent for squared returns is around 1.3:

$$\omega \approx (1.3)$$

The scaling exponent for absolute returns is around 0.8:

$$\alpha \approx 0.8$$

We empirically find a power law for mean absolute returns:

$$\mu \sim \alpha$$

The scaling exponent for long time scales:

- Heterogeneities become the most obvious.
- Time scale (frequency of interventions) is the aspect for which the location and different time horizons:
- Different people with different incentives, different geographical.
- Not only the market makers but many participants.

FROM LONG TO SHORT HORIZONS

S.F. INFORMATION FLOW
They influence the market on different time scales.

- Central banks
- International companies
- Fund managers
- Independently dealers

Categories of market participant

Informations flow asymmetric: The market effect
\[
\left(\left[ \frac{\mu^2}{L} \right]^2 (L+1) \right] \phi (L+i) \left[ \frac{\mu^2}{L} \right] \phi \right)_{\text{corr}} = \left( (L+i) \left[ \frac{\mu^2}{L} \right] \phi \right) \left[ \frac{\mu^2}{L} \right] \phi \right)_{\text{corr}} = \left\langle u \mid n \right\rangle \}
\]