Applications
- Demonstration of a software tool for deseasonalization
  - Frequency domain tools for deseasonalizing univariate high-
  - Criteria for deseasonalization methods
  - Approaches for deseasonalizing univariate data
  - Primarily with daily data

Deseasonalization of Financial Data

High-Frequency Financial Data

Weekends Included

Autocorrelation of daily absolute returns for USD/DEM, FX spot rates.

Daily Absolute Returns

Autocorrelation of

FX prices for USD/DEM and USD/JPY.
DESEASONALIZATION

APPROACHES FOR UNIVARIATE

- Volatility
  - Both are based on a daily or weekly conditional mean of
  - Weighting methods
  - Time transformation methods
- Two different groups

DESEASONALIZATION OF DAILY DATA

AUTOCORRELATION OF DAILY ABSOLUTE RETURNS

Weekends dropped

AUTOCORRELATION OF CALL ABOSOLUTE RETURNS FOR USD/DEM FX SPOT RATES

THE WEEKLY VOLATILITY PATTERN

Removal of weekly seasonality

- Disregarding public holidays breaks weekly periodicity: thus blunting

Business time (days)
TIME TRANSFORMATION METHOD OF O&A

\[
\phi\left(\frac{1}{\beta}, \lambda, \mu, \omega, \gamma, \alpha, \beta, \rho, \gamma, \alpha, \beta, \rho, \gamma\right) = \left(\frac{\mu}{\alpha^2}\right)^{\frac{\omega}{\beta^2}}
\]

where

\[
\left(\frac{\mu}{\alpha^2}\right)^{\frac{\omega}{\beta^2}} \sim \left(\frac{\mu}{\alpha^2}\right)^{\frac{\omega}{\beta^2}}
\]

conditional mean of absolute hourly returns:

\[\beta t \equiv \text{market activity related to volatility estimated by a weekly scale proportional to market activity \(a(i)\) observed conditional to the time in the week: the transformed \(\log\) of the absolute returns by} \]

Assumption (Decapers et al., 1999): the transformed \(\log\) of the absoulte returns by

---

Daily pattern of \(u^6\) modeled by polynomials of order 6 (American, East-Asian, and European component). Activity decomposed into component activities \(u_i^6\) with an empirical observed values around 0.5 are used for making

For Brownian motion, \(\theta = 0.5\).

\[
\phi^6(t) \sim ||(\lambda, \mu)|^4|^{-\delta}
\]

Exponent \(\phi^6(t)\) is related to scaling exponent of absolute returns by
Central task is modeling the seasonal volatility pattern \( \sigma^w(D) \).

\[
\sigma^w(D) = \frac{\sigma(w, \gamma, \beta)}{\mu}
\]

Equivalently, with time transformation:

\[
\frac{\sigma^w_{t+1} \sigma^w_t}{\sigma^w_t \sigma^w_{t-1}} = \sigma^w(D)
\]

Deseasonalized volatility.

**Weighting-based methods (cont.)**
**Criteria for Deseasonalization Methods**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Time Transformation</th>
<th>Quarterly Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preseason</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Postseason</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Preseason</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Postseason</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

**Weighting Methods: Volatility Pattern**

- The model:
  - and  are not in the volatility 
  - and  are similar to the scale multiplier in the model:
  - refers to the shape of the periodic cycle at time  
  - refers to the first coefficient of the periodic cycle at time  
  - refers to the second coefficient of the periodic cycle at time  
  - is a measure of the relative size of the periodic cycle at time  
  - is a measure of the relative size of the periodic cycle at time  

**Periodic GARCH**

- and time series modelling (no two-step procedure)
- The model:
- Accounting: only a single set of parameters for deseasonalizing
- Modelling of seasonality is integrated into time series modelling

---

**Notes:**
- April 30, 2010 (1998) proposed the influence
- Stochastic Windows with a state space representation
- Bollerslev and Choy (1993) proposed the influence
Financial Time Series
High-Frequency Multivariate
Adaptable Desynchronization of

Relation to Generating Processes

Modeling Requirements

Univariate Desynchronization.
Therefore this sort of time transformation is not adequate for

Stochastic:

Time transformation to individual activity-times desires.

Activity-based time scales are different for different commodities.

Recombination allowed by weighting methods.

Since time is treated in the same way for all commodities,
Symmetricity of events of different coordinates of the series
Should be considered.
ECONOMETRIC FORECASTING AND HIGH-FREQUENCY DATA

101

DAYLIGHT SAVING TIME (DST)

102

LIST OF SOME DST PERIODS

103

UK (left) US (middle) and approximate (right) DST periods

<table>
<thead>
<tr>
<th>UK (left)</th>
<th>US (middle)</th>
<th>Approximate (right) DST periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>02 Oct 2017 to 01 Oct 2018</td>
<td>02 Oct 2018 to 01 Oct 2019</td>
<td>02 Oct 2017 to 01 Oct 2018</td>
</tr>
<tr>
<td>02 Oct 2021 to 01 Oct 2022</td>
<td>02 Oct 2022 to 01 Oct 2023</td>
<td>02 Oct 2021 to 01 Oct 2022</td>
</tr>
<tr>
<td>02 Oct 2023 to 01 Oct 2024</td>
<td>02 Oct 2024 to 01 Oct 2025</td>
<td>02 Oct 2023 to 01 Oct 2024</td>
</tr>
<tr>
<td>02 Oct 2025 to 01 Oct 2026</td>
<td>02 Oct 2026 to 01 Oct 2027</td>
<td>02 Oct 2025 to 01 Oct 2026</td>
</tr>
<tr>
<td>02 Oct 2027 to 01 Oct 2028</td>
<td>02 Oct 2028 to 01 Oct 2029</td>
<td>02 Oct 2027 to 01 Oct 2028</td>
</tr>
<tr>
<td>02 Oct 2029 to 01 Oct 2030</td>
<td>02 Oct 2030 to 01 Oct 2031</td>
<td>02 Oct 2029 to 01 Oct 2030</td>
</tr>
</tbody>
</table>

104

THE INTEGRATED VOLATILITY

105

THE SEASONAL VOLATILITY

106

DECORRELATION OF QUADRATIC VARIATION
which assumes values between 0 and 1.

\[ y_j = \sum_{i=1}^{n} y_{ij} \]

where

\[ y_{ij} = \frac{u}{y_j} \]

and

\[ (i)^{th} \text{ and } (j)^{th} \text{ factor} \]

\[ (i)^{th} \text{ factor} + (j)^{th} \text{ factor} = (i+j)^{th} \text{ factor} \]

Total Relative activity:

- **Relative Market Activity**

\[ \sum_{i=1}^{n} y_{ij} \]

- Normalization condition

- Explicitly modeled

- Market Share Factors

- **Market Share Factors**

**The Weekly Volatility Pattern**

- Seasonal Volatility Pattern for FX Spot Rates
Economic forecasting and high frequency data

Activity of Market Components

Regional Market Components

Holidays Factors

\[
\begin{align*}
\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \\
\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \ln(n) \\
0 = \frac{1}{n}
\end{align*}
\]

- Share factors given as daily opening periods.
- Interest rates of market components reprice on activity.
- Tick frequency not reliable as indicator for market.
## Fuzzy and Partial Holidays

<table>
<thead>
<tr>
<th>Date</th>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/25/99</td>
<td>Christmas Day</td>
</tr>
<tr>
<td>12/31/99</td>
<td>New Year’s Eve</td>
</tr>
<tr>
<td>01/01/00</td>
<td>New Year’s Day</td>
</tr>
</tbody>
</table>

### Example: Christmas Eve

For partial holidays, if:

$$\frac{T_i - t}{T_f - t} > \theta$$

Then:

- For January 1:
  $$T_i = 12/31/99, t = 0, T_f = 1/1/00$$
  $$\frac{12/31/99 - 0}{1/1/00 - 0} = 0.5$$

For public holidays, if:

$$\frac{T_i - t}{T_f - t} = 0$$

Then:

- For New Year’s Day:
  $$T_i = 12/31/99, t = 0, T_f = 01/01/00$$
  $$\frac{12/31/99 - 0}{01/01/00 - 0} = 0$$

### Holdays (cont.)

**Weekends**

During weekends large returns occur quite often.
DESEASONALISED RETURNS

Weekend Returns

Demonstration of Desesasonalization Tool

\[
\{(\mu_{\text{week}}, \sigma_{\text{week}}^2)\} = \left\{ \mu \pm \frac{\sigma}{\sqrt{n}} \right\} \quad \text{with weekend length } n \in \mathbb{N}
\]

- Therefore define weekend volatility:
- Clutting out returns destroys consistency of return series.
- Desesasonalization result without special treatment they detoriate the activity.
- Jumps cannot be accounted for by mean weekend.

Weekends (cont.)
De-seasonalization measure

De-seasonalization

Static De-seasonalization

Dynamic De-seasonalization

Properties

\[
\frac{E[\epsilon^2] - (\bar{\epsilon})^2}{\left(\frac{1}{N}\sum_{i=1}^{N} \epsilon_i^2 \right)^{1/2}} = [\epsilon]^d
\]

Autocorrelation function of absolute returns

- Time in the week.
- Being the volatility and \( \mu \) the moving average conditional to

\[
\mu_A = \frac{[\epsilon]^d}{|\epsilon|^2}
\]

Measure of de-seasonalization quality:

De-seasonalization quality:

Of Absolute Returns

Autocorrelation Functions

De-seasonalization quality:

Weighting has to be introduced:

In order to take into account the weekends an appropriate

\[
\text{Weights} = \left( \frac{1}{N} \sum_{i=1}^{N} \text{Weights} \right) - \left( \frac{1}{N} \sum_{i=1}^{N} \text{Weights} \right)
\]

Heteroscedasticity, use of weekly conditional MAs:

To separate the mean seasonal fluctuations from the stochastic

\[
\text{Weights} = \left( \frac{1}{N} \sum_{i=1}^{N} \text{Weights} \right) - \left( \frac{1}{N} \sum_{i=1}^{N} \text{Weights} \right)
\]

To take into account time dependence:
MISSING ACTIVITY FOR USD/CHF

ACF OF DESEASONALIZED HOURLY ABS. RETURNS

APPLICATIONS

- Analysis and modeling of the dependence structure
- Gap detection