Compressed Sensing
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Suppose $x$ is an unknown vector in $\mathbb{R}^m$ (depending on context, a digital image or signal); we plan to acquire data and then reconstruct. Nominally this ‘should’ require $m$ samples. But suppose we know a priori that $x$ is compressible by transform coding with a known transform, and we are allowed to acquire data about $x$ by measuring $n$ general linear functionals – rather than the usual pixels. If the collection of linear functionals has certain properties, and we allow for a degree of reconstruction error, the size of $n$ can be dramatically smaller than the size $m$ usually considered necessary. Thus, certain natural classes of images with $m$ pixels need only $n = O(m^{1/4} \log(m))$ nonadaptive nonpixel samples for faithful recovery, as opposed to the usual $m$ pixel samples.

Underlying our results is a theoretical framework based on the theory of optimal recovery, the theory of $n$-widths, and information-based complexity. The basic results concern properties of $\ell^p$ balls in high-dimensional Euclidean space: the Gel’fand $n$-widths of such balls, new families of near-optimal subspaces for Gel’fand $n$-widths, and new algorithms for processing information derived from near-optimal subspaces.