Reconstructing Multivariate Functions from Scattered Data Efficiently

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In practice, one often faces the problem of reconstructing a multivariate function from a given, finite data set. In the simplest case, such a data set consists of data values \( f_j = f(x_j) \in \mathbb{R}, 1 \leq j \leq N \), coming from an unknown function \( f \) at certain data sites \( X = \{x_1, \ldots, x_N\} \subseteq \Omega \subseteq \mathbb{R}^d \), and interpolation is the most obvious reconstruction method. In general, the data sites are scattered over the region \( \Omega \), having no structure at all.

Approximation by positive definite kernels tries to solve this reconstruction problem by fixing a symmetric kernel \( \Phi : \Omega \times \Omega \rightarrow \mathbb{R} \). Then, the approximant is chosen from the finite space \( \{\Phi(\cdot, x_j) : x_j \in X\} \). The assumption on \( \Phi \) being positive definite leads to a well-posed problem.

In this talk I will focus on the following topics:

- error estimates in Sobolev spaces for reconstruction processes from scattered data,
- the construction of a fast reconstruction and evaluation algorithm,
- examples from surface reconstruction in computer-aided design and from fluid-structure-interaction in aeroelasticity.