Let $U$ denote the enveloping algebra of a complex semisimple Lie algebra.

A family of virtual (not necessarily finite dimensional) representations $\{\pi(\nu)\}_{\nu \in \Lambda}$ of $U$ is called a coherent family if for every finite dimensional module $F$ of $U$ (in the Grothendieck group)

$$\pi(\nu) \ F = \sum_{\mu \in \Delta(F)} m(\mu, F) \pi(\nu + \mu)$$

where the summation is over the weights $\Delta(F)$ of $F$ and for $\mu \in \Delta(F), m(\mu, F)$ denotes the multiplicity of $\mu$ as a weight of $F$.

We will describe an algebraic expression (at the level of a suitable Grothendieck group) which gives the quantum analogue $\bar{\pi}$ of a $U$-module $\pi$ for quantum groups $U_q$ at roots of unity, at the level of a suitable Grothendieck group. This description involves not only the original $U$-module but in addition the coherent family of (virtual) $U$-modules to which $\pi$ belongs.

The family $\{\bar{\pi}(\nu)\}_{\nu \in \Lambda}$ thus constructed is itself a coherent family of virtual representations of $U_q$ and satisfies

$$\bar{\pi}(\ell \nu'') = \bar{\pi}(\nu'') \ St$$

for any $\nu'' \in \Lambda$; here $St$ is the Steinberg representation and $\bar{\pi}(\star)$ denotes the pullback by the Frobenius map from $U_q$ to $U$. Under certain conditions one should expect that $\bar{\pi}(\nu)$ is represented in the Grothendieck group by a $U_q$-module (and not just a virtual module) for parameters $\nu$ in a cone.

We will discuss the validity of this fact in some low rank cases.