Pricing Mortgage-Backed Securities
– Continuous Time Stanton’s Model

Min DAI

Department of Mathematics
National University of Singapore

Joint with Yue-Kuen Kwok and Hong You
Outline
Outline

- Introduction
Outline

- Introduction
- Historical Work
Outline

• Introduction
• Historical Work
  ◦ Structural Model
Outline

• Introduction

• Historical Work
  ◦ Structural Model
  ◦ Reduced-form Model
Outline

- Introduction
- Historical Work
  - Structural Model
  - Reduced-form Model
  - Stanton’s Model
Outline

• Introduction

• Historical Work
  ◦ Structural Model
  ◦ Reduced-form Model
  ◦ Stanton’s Model

• Continuous-time Stanton’s Model
Outline

• Introduction

• Historical Work
  ○ Structural Model
  ○ Reduced-form Model
  ○ Stanton’s Model

• Continuous-time Stanton’s Model

• Numerical Schemes
Outline

• Introduction
• Historical Work
  ◦ Structural Model
  ◦ Reduced-form Model
  ◦ Stanton’s Model
• Continuous-time Stanton’s Model
• Numerical Schemes
• Applications to Other Derivatives
Introduction

What is mortgage?
Introduction

What is mortgage?

- a loan in order to purchase a house
Introduction

What is mortgage?

• a loan in order to purchase a house
• collateralized by the house
Introduction

What is mortgage?

• a loan in order to purchase a house
• collateralized by the house
• similar to an amortizing bond
Introduction

What is mortgage?

- a loan in order to purchase a house
- collateralized by the house
- similar to an amortizing bond
- A mortgagor has the **right** to prepay his loan at any time
Introduction

What is a mortgage-backed security?
Introduction

What is a mortgage-backed security?

• securitized product: a claim to the cash flows generated by a pool of mortgages
Introduction

What is a mortgage-backed security?

• securitized product: a claim to the cash flows generated by a pool of mortgages

• attractive yields with little or no credit risk, trade in a liquid secondary market
Introduction

What is a mortgage-backed security?

• securitized product: a claim to the cash flows generated by a pool of mortgages
• attractive yields with little or no credit risk, trade in a liquid secondary market
• dramatic growth since its inception in 70s
Introduction

What is a mortgage-backed security?

- securitized product: a claim to the cash flows generated by a pool of mortgages
- attractive yields with little or no credit risk, trade in a liquid secondary market
- dramatic growth since its inception in 70s
  - total notional amount of MBS and collateralized mortgage obligations outstanding as of 30 June 2002 $3.9 trillion
Introduction

What is a mortgage-backed security?

• securitized product: a claim to the cash flows generated by a pool of mortgages
• attractive yields with little or no credit risk, trade in a liquid secondary market
• dramatic growth since its inception in 70s
  ◦ total notional amount of MBS and collateralized mortgage obligations outstanding as of 30 June 2002
    $3.9 trillion
  ◦ total notional amount of US treasury debt
    $3.5 trillion
The Pricing of MBS

- Black-Scholes Option Pricing
The Pricing of MBS

- Black-Scholes Option Pricing
- The key point is how to deal with the prepayment behavior
Historical Work

structural models

reduced-form models
Historical Work

structural models

• optimal prepayment policy (like the optimal exercise policy of American options)

reduced-form models
Historical Work

structural models

• optimal prepayment policy (like the optimal exercise policy of American options)

• an optimal stopping problem, or a linear complementary problem

reduced-form models
Historical Work

structural models

• optimal prepayment policy (like the optimal exercise policy of American options)
• an optimal stopping problem, or a linear complementary problem

reduced-form models

• prepayment policy given according to historic data
Historical Work

structural models

• optimal prepayment policy (like the optimal exercise policy of American options)

• an optimal stopping problem, or a linear complementary problem

reduced-form models

• prepayment policy given according to historic data

• a simple linear parabolic PDE model
Structural Model
Structural Model

- assuming interest rate follows CIR model in the risk-neutral world

\[ dr = \kappa(\mu - r)\,dt + \sigma \sqrt{r}\,dz \]
Structural Model

- assuming interest rate follows CIR model in the risk-neutral world

\[ dr = \kappa (\mu - r) dt + \sigma \sqrt{r} dz \]

- mortgagors prepay whenever \( V = \varphi(t) \)
Structural Model

- assuming interest rate follows CIR model in the risk-neutral world

\[ dr = \kappa (\mu - r) dt + \sigma \sqrt{r} dz \]

- mortgagors prepay whenever \( V = \varphi(t) \)

- basic structure model

\[
\begin{cases}
V_t + \frac{1}{2} \sigma^2 r V_{rr} + \kappa (\mu - r) V_r - r V + C(t) \geq 0 \\
V \leq \varphi(t) \\
either must take equality at any point \((r, t) \in [0, \infty) \times [0, T]\) \\
with boundary and initial conditions
\end{cases}
\]
Structural Model

Structural Model


- suboptimal prepayment: some mortgagors prepay due to exogenous reasons, e.g. migration, divorce or purchase of a better house
Structural Model


- suboptimal prepayment: some mortgagors prepay due to exogenous reasons, e.g. migration, divorce or purchase of a better house

- exogenous prepayment governed by Poisson process $dJ$, with density $\lambda(r, t)$

\[
\begin{align*}
    P(dJ = 0) &= 1 - \lambda \delta t, \quad \text{no suboptimal prepayment occurs} \\
    P(dJ = 1) &= \lambda \delta t, \quad \text{suboptimal prepayment occurs}
\end{align*}
\]
Structural Model

Dunn and McConnell model

\[
\begin{cases}
V_t + \frac{1}{2} \sigma^2 r V_{rr} + \kappa (\mu - r)V_r - rV + C(t) + \lambda(r, t)[\varphi(t) - V] \geq 0 \\
V(r, t) \leq \varphi(t)
\end{cases}
\]

either must take equality at any point \((r, t) \in [0, \infty) \times [0, T]\)
with boundary and initial conditions

compare with most basic structural model
Structural Model

drawbacks
Structural Model

drawbacks

• theoretical MBS value bounded from above $V(r, t) \leq \varphi(t)$
Structural Model

drawbacks

- theoretical MBS value bounded from above \( V(r, t) \leq \varphi(t) \)
- predicted rate of prepayment does not match observation
Structural Model

drawbacks

• theoretical MBS value bounded from above $V(r, t) \leq \varphi(t)$
• predicted rate of prepayment does not match observation

major reason
Structural Model

drawbacks

• theoretical MBS value bounded from above $V(r, t) \leq \varphi(t)$
• predicted rate of prepayment does not match observation

major reason

• mortgagors may not prepay even when it is optimal
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)

- empirical model
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)

- empirical model
- prepayment policy given
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)

- empirical model
- prepayment policy given
- linear PDE model
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)

- empirical model
- prepayment policy given
- linear PDE model
- Monte Carlo simulation easy to implement
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)

- empirical model
- prepayment policy given
- linear PDE model
- Monte Carlo simulation easy to implement

drawbacks
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)

- empirical model
- prepayment policy given
- linear PDE model
- Monte Carlo simulation easy to implement

Drawbacks

- do not explain the true underlying process
Reduced-form Model

First proposed by Schwartz and Torous (1989, Journal of Finance)

- empirical model
- prepayment policy given
- linear PDE model
- Monte Carlo simulation easy to implement

drawbacks

- do not explain the true underlying process
- may not perform well out-of-sample
Stanton’s Model (1995)
Stanton’s Model (1995)

- a breakthrough among structural models
Stanton’s Model (1995)

• a breakthrough among structural models
• key assumption: random endogenous prepayment
Stanton’s Model (1995)

- a breakthrough among structural models
- key assumption: \textbf{random} endogenous prepayment
- endogenous prepayment governed by another independent Poisson process, with density \( \rho(r, t) \)

\[
\begin{align*}
P(\text{no endogenous prepayment decision}) &= 1 - \rho \delta t, \\
P(\text{endogenous prepayment decision}) &= \rho \delta t
\end{align*}
\]

when

\( V(r, t) > \varphi(r, t) \)
Stanton’s Model (1995)

- a breakthrough among structural models
- key assumption: **random** endogenous prepayment
- endogenous prepayment governed by another independent Poisson process, with density $\rho(r, t)$

\[
\begin{cases}
    P(\text{no endogenous prepayment decision}) = 1 - \rho \delta t, \\
    P(\text{endogenous prepayment decision}) = \rho \delta t
\end{cases}
\]

when

\[ V(r, t) > \varphi(r, t) \]

- in summary, probability of total prepayment

\[
P(\text{prepayment}) = \begin{cases} 
    P_e = \lambda \delta t & V \leq \varphi \\
    P_r = (\lambda + \rho) \delta t & \text{otherwise}
\end{cases}
\]
Stanton’s Model
Stanton’s Model

• discretize time \([0, T]\) into \(N\) steps, \(\delta t = T/N\)
Stanton’s Model

- discretize time $[0, T]$ into $N$ steps, $\delta t = T/N$
- each step, unprepaid mortgage value $M_u^n$ is obtained by solving

$$\frac{1}{2}\sigma^2 r V_{rr} + [\kappa (\mu - r)] V_r + V_t - rV + C = 0$$
Stanton’s Model

- discretize time $[0, T]$ into $N$ steps, $\delta t = T/N$
- each step, unprepaid mortgage value $M^n_u$ is obtained by solving

$$\frac{1}{2} \sigma^2 r V_{rr} + [\kappa (\mu - r)] V_r + V_t - r V + C = 0$$

- mortgage value at this step given by

$$M^n = \begin{cases} (1 - P_e) M^n_u + P_e \varphi & M^n_u \leq \varphi \\ (1 - P_r) M^n_u + P_r \varphi & \text{otherwise} \end{cases}$$
Stanton’s Model

• discretize time $[0, T]$ into $N$ steps, $\delta t = T/N$

• each step, unprepaid mortgage value $M^n_u$ is obtained by solving

$$\frac{1}{2}\sigma^2 r V_{rr} + [\kappa(\mu - r)]V_r + V_t - rV + C = 0$$

• mortgage value at this step given by

$$M^n = \begin{cases} 
(1 - P_e)M^n_u + P_e\varphi & M^n_u \leq \varphi \\
(1 - P_r)M^n_u + P_r\varphi & \text{otherwise}
\end{cases}$$

• repeat these steps until $t_0 = 0$
Stanton’s Model

Other features

• mortgage liability vs asset
Stanton’s Model

Other features

• mortgage liability vs asset
• heterogenous transaction cost
Stanton’s Model

Other features

- mortgage liability vs asset
- heterogenous transaction cost

But Stanton’s Model is only a numerical algorithm.
New Model
New Model

- continuous time version of Stanton’s model
New Model

- continuous time version of Stanton’s model
- inherit all features of Stanton’s model
New Model

- continuous time version of Stanton’s model
- inherit all features of Stanton’s model
- **random** exogenous and endogenous prepayment modelled by two independent Poisson processes

\[
\begin{align*}
    dJ &= \begin{cases} 
        0, & \text{exogenous prepayment does not occur} \\
        1, & \text{exogenous prepayment occurs}
    \end{cases} \\
    dK &= \begin{cases} 
        0, & \text{mortgagor makes no endogenous prepayment decision} \\
        1, & \text{mortgagor makes endogenous prepayment decision}
    \end{cases}
\end{align*}
\]

and

\[
\begin{align*}
    E(dJ) &= P(dJ = 1) = \lambda dt \\
    E(dK) &= P(dK = 1) = \rho dt
\end{align*}
\]
New Model

With interest rate governed by CIR model,

\[
dM = [M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r + C(t)] dt + \sigma \sqrt{r M_r} dz \\
+ [\varphi(t) - M] dJ + \min(\varphi(t) - M, 0) dK
\]
New Model

With interest rate governed by CIR model,

\[ dM = \left[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r + C(t) \right] dt + \sigma \sqrt{r M_r} dz \]

\[ + [\varphi(t) - M] dJ + \min(\varphi(t) - M, 0) dK \]

Take expectation \( E(dM) = r M dt \)
New Model

With interest rate governed by CIR model,

$$dM = \left[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r + C(t) \right] dt + \sigma \sqrt{r} M_r dz$$

$$+ [\varphi(t) - M] dJ + \min(\varphi(t) - M, 0) dK$$

Take expectation $E(dM) = r M dt$

Therefore, we derive the model

$$\begin{cases}
M_t + \frac{1}{2} \sigma^2 r M_{rr} + [\kappa (\mu - r)] M_r - r M + C(t) \\
+ \lambda [\varphi(t) - M] + \rho \min(\varphi(t) - M, 0) = 0
\end{cases}$$

with boundary and initial conditions
New Model

With interest rate governed by CIR model,

\[ dM = \left[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r + C(t) \right] dt + \sigma \sqrt{r M_r} dz + [\varphi(t) - M] dJ + \min(\varphi(t) - M, 0) dK \]

Take expectation \( E(dM) = r M dt \)

Therefore, we derive the model

\[
\begin{cases}
M_t + \frac{1}{2} \sigma^2 r M_{rr} + [\kappa (\mu - r)] M_r - r M + C(t) \\
+ \lambda [\varphi(t) - M] + \rho \min(\varphi(t) - M, 0) = 0
\end{cases}
\]

with boundary and initial conditions

compare with Dunn and McConnell model
New Model

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]
New Model

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- nonlinear parabolic partial differential equation
New Model

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- nonlinear parabolic partial differential equation
- does not impose \( \varphi(r, t) \) as the upper bound
New Model

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- nonlinear parabolic partial differential equation
- does not impose \( \varphi(r, t) \) as the upper bound
- Stanton’s numerical scheme is shown to be consistent with the continuous time model
New Model

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- nonlinear parabolic partial differential equation
- does not impose \( \varphi(r, t) \) as the upper bound
- Stanton’s numerical scheme is shown to be consistent with the continuous time model
- provides a financial explanation for penalty approximation for variational inequality problem (see Forsyth and Vetzal, 2002)
Numerical Scheme
Numerical Scheme

- Stanton’s numerical algorithm is first order

\[ \epsilon \sim O(\Delta t + \Delta y) \]
Numerical Scheme

- Stanton’s numerical algorithm is first order
  \[ \varepsilon \sim O(\Delta t + \Delta y) \]

- two numerical schemes are devised to achieve
  \[ \varepsilon \sim O(\Delta t^2 + \Delta y^2) \]
Numerical Scheme

- Stanton’s numerical algorithm is first order
  \[ \varepsilon \sim O(\Delta t + \Delta y) \]

- Two numerical schemes are devised to achieve
  \[ \varepsilon \sim O(\Delta t^2 + \Delta y^2) \]
  - Crank-Nicolson scheme with generalized Newton iteration
Numerical Scheme

• Stanton’s numerical algorithm is first order

$$\varepsilon \sim O(\Delta t + \Delta y)$$

• two numerical schemes are devised to achieve

$$\varepsilon \sim O(\Delta t^2 + \Delta y^2)$$

  ◦ Crank-Nicolson scheme with generalized Newton iteration
  ◦ hybrid scheme using Crank-Nicolson and Adam-Bashforth
Numerical Scheme 1

Crank-Nicolson scheme with generalized Newton iteration

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C \]

\[ + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]
Numerical Scheme 1

Crank-Nicolson scheme with generalized Newton iteration

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C \]

\[ + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- all terms are expanded at point \((y_i, t_{n+\frac{1}{2}})\) to obtain

\[ AM^n = BM^{n+1} + \eta^{n+\frac{1}{2}} + \rho \min\{\varphi^{n+\frac{1}{2}} - \frac{1}{2}(M^{n+1} + M^n), 0\} \]
Numerical Scheme 1

Crank-Nicolson scheme with generalized Newton iteration

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C \]
\[ + \lambda (\phi - M) + \rho \min(\phi - M, 0) = 0 \]

- all terms are expanded at point \((y_i, t_{n+\frac{1}{2}})\) to obtain

\[ AM^n = BM^{n+1} + \eta^{n+\frac{1}{2}} + \rho \min\{\phi^{n+\frac{1}{2}} - \frac{1}{2} (M^{n+1} + M^n), 0\} \]

- use generalized Newton method

\[ f(x^{(k+1)}) = f(x^{(k)}) + f'(x^{(k)})(x^{(k+1)} - x^{(k)}) \]
Numerical Scheme 1

Crank-Nicolson scheme with generalized Newton iteration

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C \]
\[ + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- all terms are expanded at point \((y_i, t_{n+\frac{1}{2}})\) to obtain

\[ AM^n = BM^{n+1} + \eta^{n+\frac{1}{2}} + \rho \min\{\varphi^{n+\frac{1}{2}} - \frac{1}{2}(M^{n+1} + M^n), 0\} \]

- use generalized Newton method

\[ f(x^{(k+1)}) = f(x^{(k)}) + f'(x^{(k)})(x^{(k+1)} - x^{(k)}) \]

- perform iteration at each step
Numerical Scheme 2

hybrid scheme using Crank-Nicolson and Adam-Bashforth

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C \]
\[ + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]
Numerical Scheme 2

hybrid scheme using Crank-Nicolson and Adam-Bashforth

\[ M_t + \frac{1}{2} \sigma^2 r M_{rr} + \kappa (\mu - r) M_r - r M + C \]
\[ + \lambda (\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- nonlinear term expanded explicitly (Adam-Bashforth) to obtain

\[ A M^n = B M^{n+1} + \eta^{n+\frac{1}{2}} + \rho \min\{\varphi^{n+\frac{1}{2}} - \frac{1}{2}(3M^{n+1} - M^{n+2}), 0\} \]
Numerical Scheme 2

hybrid scheme using Crank-Nicolson and Adam-Bashforth

\[ M_t + \frac{1}{2}\sigma^2 r M_{rr} + \kappa(\mu - r)M_r - rM + C + \lambda(\varphi - M) + \rho \min(\varphi - M, 0) = 0 \]

- nonlinear term expanded explicitly (Adam-Bashforth) to obtain

\[ AM^n = BM^{n+1} + \eta^{n+\frac{1}{2}} + \rho \min\{\varphi^{n+\frac{1}{2}} - \frac{1}{2}(3M^{n+1} - M^{n+2}), 0\} \]

- no iteration required
**Numerical Result 1**

<table>
<thead>
<tr>
<th></th>
<th>Stanton’s Scheme</th>
<th></th>
<th></th>
<th>Stanton vs Crank-Nicolson with Newton Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \varepsilon_1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N_t )</td>
<td>( N_y )</td>
<td>( \varepsilon_1 )</td>
<td>ratio</td>
<td>( \varepsilon_1 )</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>127.513522</td>
<td>2.0</td>
<td>2.413700</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>64.293427</td>
<td>2.0</td>
<td>0.588450</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>32.275461</td>
<td>2.0</td>
<td>0.146250</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
<td>16.169679</td>
<td>2.0</td>
<td>0.036505</td>
</tr>
<tr>
<td>800</td>
<td>800</td>
<td>8.092824</td>
<td>2.0</td>
<td>0.009120</td>
</tr>
<tr>
<td>1600</td>
<td>1600</td>
<td>4.048412</td>
<td>2.0</td>
<td>0.002277</td>
</tr>
<tr>
<td>3200</td>
<td>3200</td>
<td>2.024710</td>
<td>2.0</td>
<td>0.000566</td>
</tr>
<tr>
<td>6400</td>
<td>6400</td>
<td>1.012484</td>
<td>2.0</td>
<td>0.000139</td>
</tr>
<tr>
<td>12800</td>
<td>12800</td>
<td>0.506278</td>
<td>2.0</td>
<td>0.000032</td>
</tr>
</tbody>
</table>

Stanton vs Crank-Nicolson with Newton Iteration
### Numerical Result 2

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>$N_y$</th>
<th>$\varepsilon_1$</th>
<th>Stanton’s Scheme</th>
<th>$\varepsilon_1$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>50</td>
<td>57.449132</td>
<td></td>
<td>0.670530</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>27.242231</td>
<td>2.1</td>
<td>0.158820</td>
<td>4.2</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>13.468115</td>
<td>2.0</td>
<td>0.038450</td>
<td>4.1</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
<td>6.705991</td>
<td>2.0</td>
<td>0.009536</td>
<td>4.0</td>
</tr>
<tr>
<td>800</td>
<td>800</td>
<td>3.346585</td>
<td>2.0</td>
<td>0.002374</td>
<td>4.0</td>
</tr>
<tr>
<td>1600</td>
<td>1600</td>
<td>1.671741</td>
<td>2.0</td>
<td>0.000591</td>
<td>4.0</td>
</tr>
<tr>
<td>3200</td>
<td>3200</td>
<td>0.835489</td>
<td>2.0</td>
<td>0.000147</td>
<td>4.0</td>
</tr>
<tr>
<td>6400</td>
<td>6400</td>
<td>0.417650</td>
<td>2.0</td>
<td>0.000036</td>
<td>4.1</td>
</tr>
<tr>
<td>12800</td>
<td>12800</td>
<td>0.208801</td>
<td>2.0</td>
<td>0.000008</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Stanton vs Hybrid Scheme
## Numerical Result

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Order</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stanton’s scheme</td>
<td>1</td>
<td>no</td>
</tr>
<tr>
<td>CN with Newton Iteration</td>
<td>2</td>
<td>yes</td>
</tr>
<tr>
<td>Hybrid scheme</td>
<td>2</td>
<td>no</td>
</tr>
</tbody>
</table>
## Numerical Result

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Order</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stanton’s scheme</td>
<td>1</td>
<td>no</td>
</tr>
<tr>
<td>CN with Newton Iteration</td>
<td>2</td>
<td>yes</td>
</tr>
<tr>
<td>Hybrid scheme</td>
<td>2</td>
<td>no</td>
</tr>
</tbody>
</table>

Conclusion: Hybrid scheme is most suitable to solve this model.
Applications

- CN scheme with Newton iteration can be used to solve penalty approximation for American options
Applications

• CN scheme with Newton iteration can be used to solve penalty approximation for American options

• model other derivatives: convertible bonds, callable warrants, ...
Recap
Recap

• a new continuous time structural model for MBS
Recap

• a new continuous time structural model for MBS
• second order numerical schemes
Recap

• a new continuous time structural model for MBS
• second order numerical schemes
• applicable to other derivatives
Recap

• a new continuous time structural model for MBS
• second order numerical schemes
• applicable to other derivatives

Thank you